Effort Provision in Peer Groups *

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Abstract

We study a model in which individuals, that are heterogeneous along a single dimension capturing productivity, choose which of two available groups to join and how much costly effort to exert within their chosen group. On the one hand, individuals like to be in groups where others' average performance is high (global quality). On the other hand, individuals are concerned with their ranking with respect to their peers' average performance (local standing). Nash equilibrium efforts are such that the higher the individual's productivity the higher her private outcome. In contrast, it is not necessarily the case that highly productive individuals exert more effort. When social welfare is measured as the sum of individual utilities, Nash equilibrium efforts are never efficient and whether they are higher or lower than efficient efforts depends on the strength of global quality versus local standing concerns. Moreover, stable partitions of society into groups may either resemble grouping by productivity or productivity mixing. In contrast, efficient partitions must always exhibit grouping by productivity.

Keywords: peer groups, segregation, mixing, effort choices, welfare

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1 Introduction

According to the theory of local comparisons (Festinger, 1954) individuals have an innate desire to evaluate themselves and do so through comparisons with others. Nowadays, it is widely accepted that individuals compare themselves with others in their local reference group and that such comparisons determine, at least partially, their happiness.[1] Falling behind others may be a source of pain and being above others quite often provides benefits.[2] Apart from these local comparison concerns, individuals also like to belong to high-quality groups, where members are successful, because of the positive influence that such members usually have on others.

Heath (1993) emphasizes that one of the most important determinants of university selection by students is the status or prestige of such an institution, but also, that students are concerned about their academic standing within their circle of classmates and friends. The annual Survey of Admitted Students conducted by The National Research Center for College and University Admissions in the United States reveals that, for more than 60% of the students, school academic strength/quality is a key determinant of their enrollment decisions, and that concerns about poor performance are one of the top reasons as to why students decline enrollment offers.[3] In a similar spirit, Nolfi (1979) indicates that the attractiveness of educational alternatives for an individual first increases with the average quality of enrolled students and then decreases when such an average quality is above the ability of the individual in question.

This paper examines how concerns for local standing and group quality determine the formation of groups by individuals and the provision of effort in such groups.

We analyze a full information game that is composed of two stages. In the first stage, individuals simultaneously decide which of the two available groups to join, with the restriction that each individual can belong to at most one of the groups. Individuals then learn how groups are composed and in the second stage decide how much effort to exert within their group. The effort then translates into a private outcome, that is, a grade in an exam or the number and quality of publications in a research department. Finally, payoffs are realized.

[1] See Frank (1985) and Frank (2013) for an illuminating analysis of the effect of status considerations on a wide range of economic and social dimensions such as salaries or health. See also Dijkstra et al. (2008) and Dumas et al. (2005) for evidence of how local comparisons occur in the classroom. Ferrer-i Carbonell (2005) shows that the self-reported life satisfaction of West Germans is affected by the income of individuals in their reference group.

[2] Choi et al. (2022) document how the prevalent cultural norm of Singapore, Kiasu, commonly translated as Fear of Losing Out, generates a constant concern among students about not keeping up with others.

[3] See https://encoura.org/mind-gap-targeting-student-concerns-yield/ and https://encoura.org/mind-gap-targeting-student-concerns-yield/.

We assume that individuals are heterogeneous and that this heterogeneity materializes along a single dimension that reflects private productivity.

We also assume that individuals' utility can be separated into two components: a private component and a social component. The private component reflects the utility that accrues to an individual due only to her productivity and to her exerted effort whereas the social component reflects the utility that accrues to an individual due to her social concerns. Such a social component is composed of two elements: on the one hand, individuals care about the quality of the group they belong to and, on the other hand, they care about how they rank relative to others' average performance within their own group.

The importance of group quality is justified by the positive effects that peers have on individuals' achievements and therefore we consider a framework where complementarities play a role, specifically, from the point of view of any individual an increase in others' performance triggers an upward shift in that individual's effort. Prominent models of individual choice within social networks (Ballester et al., 2006; Ushchev and Zenou, 2020; Horváth, 2025) incorporate complementarities in analogous fashions. Whereas in these models individuals' choices depend on the choices made by others to whom they are linked, we model groups instead of networks, and thus pairwise relations are not necessarily specified. Further, we assume that to make their choices, individuals do not necessarily rely on information regarding each other's effort choices, which may be very demanding. Instead, we consider that individuals have some information regarding the quality of their group, specifically, others' average performance. We believe that this is a natural assumption in contexts where the emphasis is on group performance as a whole rather than on pairwise relations.

Regarding local standing we simply posit that individuals are better off the higher they rank relative to others' average performance within their own group. In the area of education, there are several reasons for students to care about their relative position: it may be that better positions provide future benefits (Elsner and Isphording, 2017) or positively affect admissions at higher levels of education (Grau, 2018).[4]

We analyze partitions of the society into two groups and conceive a partition as stable when no individual has unilateral incentives to move from her current group to the alternative group. Notice that, as individuals' types are defined along a single dimension capturing productivity, groups can only consist of one (and thus consecutive) interval of productivity values or the union of non-consecutive intervals of productivity values.[5]

[4] In a more anecdotal fashion, performing well in a research department may grant professionals access to non-pecuniary benefits such as corner offices or benefiting from the possibility of sabbatical periods.

[5] Briefly, a group is consecutive if for any pair of individuals' types belonging to it, all the types in between also belong to it.

As is typical in a large proportion of group and network formation games stable architectures are multiple in nature but, despite this potential multiplicity, we provide (Proposition 1) characteristics that groups in stable partitions must exhibit, thus limiting the number of such stable partitions.

We consider effort choices that constitute a Nash equilibrium of the second stage of the game and conclude that such an equilibrium exists provided that individuals do not place excessive weight on others' choices so that there is no escalation of efforts (Lemma 1). Nash equilibrium efforts are such that the private product individuals obtain, namely, productivity times effort, preserves the order of the exogenously given private productivities, in other words, more productive individuals produce more. However, it does not necessarily follow that more productive individuals exert more effort since this decision depends on who are the peers surrounding them. The mechanism behind this result is that when highly productive individuals are surrounded by peers with low productivity the complementarities between those peers' average performance (which is low) and the effort exerted by highly productive individuals are hardly exploited and thus, in response, the effort exerted by highly productive individuals is relatively small. The contrary happens to individuals of low productivity as they face a high average performance by their (highly productive) peers.

We finally offer insights on how group configurations and efforts should be designed if the interest is to maximize social welfare. When social welfare is defined as the sum of individual utilities, a necessary condition to maximize social welfare is that within each group individuals exert what we call efficient efforts. Efficient efforts and Nash equilibrium efforts never coincide (Lemma 4), specifically, effort choices could be simultaneously efficient and a Nash equilibrium whenever all the individuals exert the same level of effort, but this symmetric proposal cannot be implemented either as efficient efforts or as a Nash equilibrium.

The conflict between Nash equilibrium and efficient efforts has its roots in the different ways social concerns are incorporated. Nash equilibrium efforts incorporate both, concerns for local standing and group quality whereas from the point of view of efficiency concerns for local standing do not play a role. The reason is that, in the aggregate, the utility gains of individuals who are above others' average performance offset the utility losses of those who fall below. Thus, only group quality affects social welfare, in particular, in the aggregate each individual's private product would positively affect all the remaining individuals in her group via average performance. This positive externality is incorporated in efficient efforts but it is not internalized by individuals when they choose effort in a decentralized way.

These observations lead to the conclusion that increasing the importance that individuals grant to group quality and decreasing the importance they grant to local standing raise efficient efforts with respect to Nash equilibrium efforts.

With respect to the configuration of partitions, we show that social welfare is maximal when individuals are organized in consecutive groups (Proposition 3 and Proposition 4). When social welfare is the sum of individual utilities this result follows from the fact that the sum of individual utilities is essentially the sum of individuals' private product (Proposition 2) and, crucially: (i) each individual's efficient effort increases with others' productivity and (ii) the more productive individuals are the more sensitive they are to others' productivity. When, alternatively, social welfare is defined as the sum of individual efforts, and the individuals within a group are assumed to play Nash equilibrium efforts so that only group configurations can be manipulated, it is also the case that efficient partitions consist of consecutive groups and the reason is analogous to the one posed above.

We close the section devoted to social welfare with an extensive analysis of whether efficient partitions can also be stable (see the discussion in Section 5).

1.1 Our contribution

1. We contribute to the literature that investigates the role of status concerns by naturally describing a process by which individuals form groups to engage in a strategic choice of effort afterward. Effort affects group quality, which confers global status, and also the individuals' position relative to others' average performance, which confers local status. While in our model global and local status are endogenous, to the best of our knowledge previous literature (as it will be largely described below) either considers fixed social structures within which individuals make strategic choices, such as consumption or effort, or individuals only choose their social circle. We thus aim to build in the direction of reconciling these two approaches.

2. We also aim to feed the discussion of how status concerns affect group formation in environments other than firms.[6] We have in mind decisions such as accepting offers at universities or the formation of study groups by students. Our analysis may inform the understanding of how individuals' private productivities are the key drivers of the incentives to join groups of varying quality. In particular, we rationalize the emergence of non-consecutive groups, featuring a mixing of individuals' productivities, and not only consecutive groups. That is in contrast with the pervasive emergence of segregative outcomes in group formation models within the strand of literature studying jurisdictions and the provision of public goods (see Baccara and Yariv (2013) and Jehiel and Scotchmer (2001) and the references therein).

The works by Pack and Pack (1977) for Pennsylvania and Persky (1990) for the Chicago metropolitan area conclude that communities appear to be more heterogeneous than the

[6] See Gola (2024) for a study on occupational sorting under social status concerns.

well-known Tiebout model predicts. In a similar vein, Stein (1987) documents little sorting across different dimensions (income, occupation, education) in the majority of the states in the United States. For the Boston metropolitan area, Epple and Platt (1998) document how the income of the wealthiest households in a jurisdiction of low average income exceeds the income of the poorest households in a jurisdiction of high average income.[7]

Also, anecdotal evidence suggests that students do not always attend the most selective college (where students may be, as a consequence of the selection, of high productivity) that has admitted them and that the desire to be a big fish in a small pond (a better student surrounded by relatively worse students) may guide this choice.[8] The model offers results in line with this evidence, that is, stable partitions may arise in which some individuals prefer to be surrounded by lowly productive peers instead of belonging to groups in which individuals are highly productive. The basic mechanism is that for an individual who is evaluating whether to move to an alternative group the fact that individuals in such a group are highly productive and perform, on average, better than individuals in her current group does not compensate her if she largely falls below others' average performance in such a group.

3. We shed light on the relationship between effort choices and group formation in the presence of status concerns. In our model more productive individuals obtain higher private outcomes, but it is not always the case that they exert more effort, as stated above, the level of effort exerted depends on the productivity of that individual's peers.

4. We offer an extensive analysis of social welfare and provide relevant insights on how to design groups in order to maximize it.

The rest of the paper unfolds as follows. Section 2 provides further literature connections. Section 3 presents the model and the equilibrium concept. Section 4 presents the equilibrium analysis. Section 5 is devoted to the social welfare analysis and Section 6 concludes. The Appendix in Section 7 contains a discussion on additional aspects such as: the role of externalities between groups (Subsection 7.1), the case of incomplete information about individual productivities (Subsection 7.2), the case in which there are more than two groups (Subsection 7.3) and additional discussions on the existence of stable partitions and efficient efforts (and how to restore them) respectively in Subsection 7.4 and Subsection 7.5. The final Subsection 7.6 contains the technical proofs.

[7] Also, see Staab (2024) for a discussion on these lines.

[8] See https://www.moorecollegedata.com/post/the-less-prestigious-college-choice. Additionally, the empirical analysis by Cakir (2019) for the political system in Turkey also suggests that resourceful politicians prefer to be part of less prominent parties, where they are more influential, whereas politicians with little assets prefer to benefit from well-established parties

2 Further literature connections

This paper is closely related to the literature that focuses on status concerns. To the best of our knowledge, Damiano et al. (2010) and Staab (2024) are the most closely related papers as both of them incorporate individuals' concerns for group quality and local standing. Below we discuss how such models differ from the one we present here.

Damiano et al. (2010) consider a model in which individuals choose between two organizations of fixed capacity and derive utility from the mean quality of an organization as well as from their ranking within such an organization. Contrary to our case, apart from choosing an organization, no additional choice is made by individuals. Furthermore, the authors consider a many-to-one matching model in which the resulting equilibrium consists of two overlapping intervals of individuals' types. In contrast to our results, perfect segregation of the individuals into groups is not a necessary characteristic of efficient configurations.

Staab (2024) considers a model in which individuals observe prices for group membership and must decide group belonging and the level of engagement within their group. In contrast to our case, a key assumption is that higher types value group quality more. Additionally, the level of engagement determines how much an individual benefits from a group, but it has no strategic implications. In our case, efforts result from a strategic interaction of group members and such efforts determine local standing and group quality. The research questions are also different, whereas our interest is in the relationship between strategic effort choices and group formation Staab (2024) analyzes which groups can be formed and which ones might be offered by an institution, such as a monopolist or a competitive market.

Within the line of research studying the role of local standing of individuals embedded in networks, López-Pintado and Meléndez-Jiménez (2021) consider a dynamic model of random networks in which individuals derive extra utility when their performance is above a comparison threshold that measures peers' performance, as in our case. In contrast to our case, the authors primarily consider homogeneous agents and do not investigate the role of group quality. The authors' main research question, namely the role of competitiveness in large societies, is also different from ours.

Ghiglino and Goyal (2010) and Immorlica et al. (2017) analyze the impact of local comparisons on the choices made by individuals embedded in exogenously given social networks. In contrast, in our case group belonging is endogenously determined. The research questions in these two papers are also different from our main focus. Ghiglino and Goyal (2010) study the implications of allowing local comparisons in a general equilibrium model and thus they are concerned with how equilibrium prices and allocations are affected by such comparisons.

Immorlica et al. (2017) consider that only upward comparisons are of importance and within such a framework they analyze the role of cohesion on the equilibrium outcomes.

Bramoullé and Ghiglino (2022) analyze the role of loss aversion in consumption in networks, a research question that greatly differs from our approach. They find that, in some circumstances, consumers choose the same level of consumption to avoid status losses. In our model, it is never the case that individuals' private products are the same within a group.

Ushchev and Zenou (2020) study a model in which individuals have preferences for conformity and interact on a fixed network. The authors characterize the Nash equilibrium of individual actions and also study efficient actions. In an extension of the model, the authors show that if individuals also have the option to choose their friends (endogenous network), then the only pairwise Nash stable network is the complete one or the homophilic network, in which individuals relate only to others of the same type. In contrast, the current model allows for the possibility of extreme homophilic relations but also heterophilic ones in which a mixing of productivities emerges.

There is also abundant literature studying the formation of groups. We mention here some prominent pieces of research:

Watts (2007) studies a model in which individuals, who care either about local comparisons or group quality, but not both simultaneously (contrary to our case), decide which group to join. Beyond group belonging, there are no additional decisions made by individuals. Some of the central research questions are also different, in particular, the author analyzes what happens to stable partitions when new locations are added.

Milchtaich and Winter (2002) consider a model of group formation with fixed groups in which individuals have preferences for joining the group with individuals similar to themselves, in contrast, we do not consider such homophilic preferences. Their research question is also different from our main focus as the authors are mainly concerned with the conflict between stability and efficiency.

Nguyen et al. (2020) analyze stability and efficiency within a model in which individuals may join multiple social groups. In contrast to our proposal, the authors consider a utility function which resembles the one proposed in the connections model by Jackson and Wolinsky (1996), in particular, individuals are heterogeneous in the cost of joining groups and get the intrinsic value of each particular group they belong to.

Morelli and Park (2016) study the formation of coalitions by heterogeneous agents through a cooperative game in which agents care about the power of their coalition and their ranking within it. Contrary to our case, the number of groups is endogenous to the model and some of the research questions, such as how the division of the surplus determines the structure of coalitions, are different from our main focus.

As briefly advanced, the local public good literature is also related to our proposal as it analyzes the formation of jurisdictions where a local public good is to be produced (Wooders, 1980; Greenberg and Weber, 1986; Gravel and Thoron, 2007). The coalition formation literature (Bogomolnaia and Jackson, 2002; Banerjee et al., 2001) also shares some common aspects, as it essentially studies group formation. The focus of these papers is on the role of different stability notions in the context of hedonic coalition formation games. Another paper that addresses group formation in a public group provision game is Ahn et al. (2008).

3 The model

Let $\mathcal{N}$ be a set with a population of N individuals. Each individual is labeled as $i \in \{1, 2, ..., N\}$ and is characterized by an exogenous productivity parameter $b_i \in (0, \infty)$ that defines her type. Without loss of generality, we assume that $b_1 > b_2 > ... > b_N$.

A partition of the society is a specification of two groups such that each individual belongs to exactly one group. Thus, the number of groups is fixed but the formation of these groups is endogenously determined.

The utility of an individual i in group G consists of a private component and a social component. Regarding the private component, individual i enjoys the product generated when she exerts a costly effort $e_{i,G} \in (0, \infty)$.[9] The social component consists of two aspects: the quality of the group and the individual's standing with respect to others' average performance within her group. The expression for others' average performance is given by

$$A_{i,G} = \frac{\sum_{j \neq i \in G} b_j e_{j,G}}{|G| - 1}. \tag{1}$$

We assume that the utility of individual $i \in G$ takes the form

$$u_i(e_{i,G}, e_{-i,G}) = \underbrace{b_i\, e_{i,G} - \frac{1}{2}\, e_{i,G}^2}_{\textit{private component}} + \underbrace{\underbrace{\alpha[e_{i,G}\, A_{i,G}]}_{\textit{group quality}} - \underbrace{\beta[A_{i,G} - b_i\, e_{i,G}]}_{\textit{local standing}}}_{\textit{social component}}, \tag{2}$$

where $\alpha, \beta > 0$. The utility of an individual who is the only member of her group consists of the private component

$$u_i(e_{i,\{i\}}, e_{-i,\{i\}}) = b_i e_{i,\{i\}} - \frac{1}{2} e_{i,\{i\}}^2,$$

[9] We use below the shorthand notation $e_{-i,G}$ to refer to the efforts exerted by individuals other than i.

where, for consistency, we use $e_{-i,\{i\}}$ to account for others' efforts, although in this case such efforts are zero because individual i does not have peers.

In Eq. (2) the private component consists of the private product minus the (convex) cost of effort and the social component includes concerns for both, group quality and local standing. Specifically

(i) group quality materializes in that there are complementarities between others' average performance and own productivity.[10] From the point of view of a given individual, an increase in others' average performance triggers an upward shift in her own effort. The parameter α captures the relevance of the group quality component.

(ii) local standing materializes in that individuals compare their performance with the average performance of others within their group and derive utility losses whenever they fall behind such an average performance and utility gains when they stand above. The parameter β captures the relevance of local standing.[11]

From Eq. (2) it follows that, everything else equal, higher average performance: (i) benefits individuals through improved group quality but (ii) hurts individuals via more disadvantageous local standing, thus, a trade-off emerges. As we will see below, in extreme scenarios in which an individual is highly productive only group quality matters for her decision of which group to join.

We study a two-stage game of full information with the following timing

1. Individuals simultaneously decide which group to join.
2. Individuals learn the composition of groups.
3. Individuals simultaneously choose the effort they exert in their own group.
4. Payoffs are realized according to Eq. (2).

We believe that this two-stage model matches some scenarios that may naturally emerge in real-life environments such as those in which students first decide which enrollment offer from a university to accept and afterward, how much effort to exert once they are surrounded by their peers, or those in which professionals decide which offer by institutions (universities, firms) to accept and afterward, how much effort to exert once they form part of such an institution.

We focus on pure strategy Nash equilibria and our main motivation for this choice is to

[10] As Damiano et al. (2010) state: "naturally, people desire to join organizations with high-quality members if being in the company of high-quality colleagues raises their own utility or productivity."

[11] The study by Mujcic and Frijters (2013) analyzes different dimensions of the trade-off between absolute and relative income by university students in Australia. The authors conclude that the relative comparison income model, in which individuals compare themselves to the average income in society, is the one that best accounts for the data when predicting the observed choices.

offer a (perhaps) clean prediction regarding which groups individuals join and hence whether productivity grouping or mixing takes place.[12] Beyond our aim, there is experimental research that points out that individuals tend to play pure strategies (see Friedman (1996) and also the discussion in Cartwright and Wooders (2009).[13]

Let an arbitrary partition be denoted by $\mathcal{G}$ and $\mathbb{G}$ be the set of all possible partitions that can be formed by the population $\mathcal{N}$ of size N. A strategy of an individual i consists of a pair $\{G_1, G_2\} \times e_i$, where the first component refers to the group individual i wishes to belong to and the second component is a mapping $e_i : \mathbb{G} \to \mathbb{R}_+$ such that $e_{i,G\in\mathcal{G}} \in \mathbb{R}_+$ is the effort made by individual i in group $G \in \mathcal{G}$, for a particular partition $\mathcal{G} \in \mathbb{G}$. When there is no ambiguity we simply use the shorthand notation $e_{i,G}$ to refer to the effort made by individual $i \in G \in \mathcal{G}$. A profile of effort strategies $e \equiv (e_{i,G})_{i=1,\dots,N} \in \mathbb{R}_+^{N\times|\mathbb{G}|}$ is a collection of efforts made by individuals for each partition $\mathcal{G}$ and each group $G \in \mathcal{G}$.

We study effort choices and partitions that constitute a subgame perfect equilibrium of the proposed game. In particular, we require that partitions are immune to unilateral deviations and that for each possible partition effort choices exerted by individuals in their own groups constitute a Nash equilibrium. Definition 1 and Definition 2 help to formalize these ideas.

Definition 1. (**Effort-choice subgame -Nash equilibrium-**) Fix $G \in \mathcal{G}$. Then, the effort choices $e_{i,G}$ for each $i \in G$ constitute a Nash equilibrium of the second stage of the game (effort choice subgame) whenever

$$u_i(e_{i,G}, e_{-i,G}) \geq u_i(e'_{i,G}, e_{-i,G}), \quad e'_{i,G} \neq e_{i,G}.$$

Definition 2. (**Stable partition**) Partition $\mathcal{G} = \{G_1, G_2\}$ is stable whenever for each individual $i \in G_s \in \mathcal{G}$, , $s, s' \in \{1, 2\}$ and $s' \neq s$

$$u_i(e_{i,G_s}, e_{-i,G_s}) \geq u_i(e_{i,G_{s'}\cup\{i\}}, e_{-i,G_{s'}\cup\{i\}}).$$

Two comments are in order: first, for the main results we are not considering situations in which the ability of individuals to move across groups is restricted by the consent of members in the group they wish to join.[14] We discuss on this possibility in Subsection 7.4. Second, stability only relies on the robustness to unilateral deviations and not on group deviations.

[12] In this game equilibrium existence is guaranteed, in particular, once we solve for the second stage Nash equilibrium efforts the game ultimately consists of a finite number of individuals each with a finite number of strategies, namely, group belonging, therefore in this case Kakutani's fixed point Theorem applies.

[13] Also, in dynamics contexts, there is literature pointing out the emergence of serial correlation in the use of mixed strategies, an issue that interferes with the assumption that individual choices are not predictable by opponents. For references see Walker and Wooders (2001) and Duffy et al. (2024).

[14] We thus consider the Nash stability notion in Bogomolnaia and Jackson (2002), Milchtaich and Winter (2002) and Bogomolnaia et al. (2008).

4 Equilibrium analysis

In this section, we first analyze equilibrium efforts within a given group and then proceed to the analysis of stable partitions. For the last part, we pay special attention to group configurations that resemble sorting by productivity and productivity mixing.

4.1 Exerting effort in a group

From the utility specification in Eq. (2), it is direct to assess that the optimal effort of an individual is increasing in others' average performance, in particular, for an individual $i \in G$ that faces others' average performance $A_{i,G}$ the best reply is $e_{i,G}(e_{-i,G}) = b_i(1+\beta) + \alpha A_{i,G}$.[15]

Nash equilibrium efforts exist in this context when the impact of an individual's effort on that of her peers in a group is (eventually) less than one-for-one so that there is no escalation of efforts.

To introduce the formal result let us focus on a non-singleton group G and let W be a square matrix of size $|G|$, whose largest eigenvalue is $\mu_1(W)$ and that has entries: $w_{ii} = 0$ and $w_{ij} = b_j/(|G| - 1)$, $j \neq i$. Analogously, let I be the identity matrix of size $|G|$.

Lemma 1. The matrix $[I - \alpha W]^{-1}$ is well-defined and non-negative if and only if $1 > \alpha\mu_1(W)$. Then, the effort choice subgame has a unique Nash equilibrium. In such an equilibrium $b_i\, e_{i,G} > b_j\, e_{j,G}$ for each pair of individuals i, j such that $b_i > b_j$.

The largest eigenvalue modulus captures the extent to which a change is amplified within the group. When such a modulus is sufficiently small, the impact of an individual's effort on that of her peers in a group is (eventually) less than one-for-one.

The result in Lemma 1 states that individuals' productivities predict individual outcomes, which is consistent with the literature on students' performance and the relation between cognitive skills and wages (Murnane et al., 1995; Schmitt et al., 2007; Blázquez et al., 2018). We emphasize that in contrast, it is not always the case that more productive individuals exert more effort. That is consistent with the findings by Babcock and Betts (2009) who suggest that ability and effort are positively but not perfectly correlated. The mechanism behind our result is that when highly productive individuals are surrounded by peers of relatively low productivity, the complementarities between those peers' average performance (which is low) and the effort exerted by highly productive individuals are hardly exploited and thus, in

[15] The best reply of an individual $i \in G$ would be the same regardless of the chosen indicator of others' performance in the local standing part, for instance, we may consider the private product of a particular individual. Such a best reply would be also increasing in others' average performance for any increasing transformation of such an indicator in the group quality part.

response, the effort exerted by highly productive individuals is relatively small. Note that the contrary happens to individuals of low productivity as they face a high average performance by their (highly productive) peers. Overall, this mechanism thus also emphasizes the role of friends' outcomes on individuals' performance.[16]

Remarkably consistent with the findings by Hopkins and Kornienko (2004) is that the excess of efforts made by individuals within a given group when social concerns are present leaves them ranked equally, according to private products, than when social concerns are absent. More specifically, note that for $\alpha = \beta = 0$ each individual optimally exerts effort $e_{i,G}(e_{-i,G}) = b_i$, and thus individuals' private products also preserve the ranking of individuals' productivities.

The following example illustrates the Nash equilibrium of the effort choice subgame induced in the case of two individuals that form a group.

Example 1. Let $G = \{1, 2\}$, $\alpha, \beta > 0$, and individual productivities be $b_1 > b_2$. In this case, Nash equilibrium efforts are

$$e_{1,G} = (1+\beta)\frac{b_1 + \alpha b_2^2}{1-\alpha^2 b_1 b_2}, \quad e_{2,G} = (1+\beta)\frac{b_2 + \alpha b_1^2}{1-\alpha^2 b_1 b_2}.$$

Observe that the more important the social component, via larger β or larger α, the higher the efforts exerted by both individuals. Notice also that each individual's effort is increasing in her own and others' productivity.[17] For the case $\beta = 1$, $\alpha = 0.5$ and $b_1 = 0.8 > b_2 = 0.4$ Nash equilibrium efforts are $e_{1,G} = 1.9$ and $e_{2,G} = 1.56$, and thus $b_1 e_{1,G} = 1.52 > b_2 e_{2,G} = 0.62$, as stated in Lemma 1.

4.2 Stable partitions

The discussion around stable partitions may benefit from the introduction of the following definitions.

Definition 3. **Consecutive group.** Group G is consecutive if for any pair $i, j \in G$ such that $b_i < b_j$ it follows that $k \in G$ whenever $b_i < b_k < b_j$.[18]

As individuals can be organized in groups that are either consecutive or non-consecutive, we can categorize the relation between the two groups according to the productivities of the individuals that form them.

[16] For a reference on this issue see Berndt (1999) and also the references therein.

[17] It can also be shown that the higher an individual's productivity the higher the effect that an increase in others' productivity has on her own effort, formally, for each i it holds that $\partial^2 e_{i,G}/\partial b_i \partial b_j > 0$ for $j \neq i$.

[18] Baccara and Yariv (2013) and Bogomolnaia et al. (2008) also consider the notion of consecutive groups and Greenberg and Weber (1986) consider a seminal related concept in the context of Tiebout economies.

Definition 4. **Absolute dominance.** Group G_s absolutely dominates other group $G_{s'}$, $s \neq s'$ whenever $\forall i \in G_s$ and $\forall j \in G_{s'}$ it follows that $b_i > b_j$.

This dominance relation is illustrated in the partition in Fig. 1, which is composed by consecutive groups.

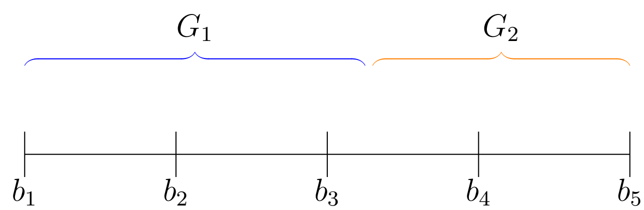

Figure 1 – A partition in which G_1 absolutely dominates G_2

In contrast, in the partitions in Fig. 2 and Fig. 3 (some of the) groups are non-consecutive, and, in a sense, more heterogeneous in their compositions than if they both were consecutive.

Non-consecutive groups relate to each other in two alternative ways. The first case is illustrated in Fig. 2, where for each individual in G_2 there is an individual in G_1 with higher productivity, and for each individual in G_1 there is an individual in G_2 with lower productivity (see Definition 5). The second case is illustrated in Fig. 3, where some individuals in G_1 there is no individual in G_2 with lower productivity.

Definition 5. **Relative dominance.** Group $G_s = \cup_{k=1}^{k'} \mathcal{I}_k$ relatively dominates other group $G_{s'} = \cup_{l=1}^{l'} \mathcal{J}_l$ whenever for each subinterval $k = l$ for each $i \in \mathcal{I}_k$ and for each $j \in \mathcal{I}_l$ it follows that $b_i > b_j$.

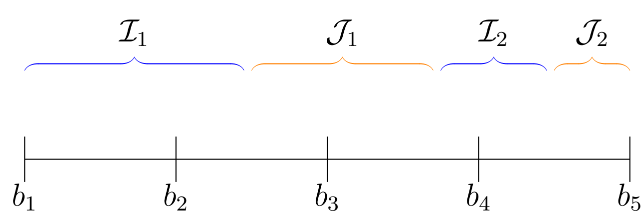

Figure 2 – A partition in which $G_1 = \mathcal{I}_1 \cup \mathcal{I}_2$ relatively dominates $G_2 = \mathcal{J}_1 \cup \mathcal{J}_2$

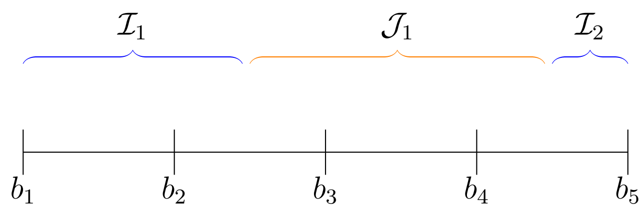

Figure 3 – A partition in which $G_1 = \mathcal{I}_1 \cup \mathcal{I}_2$ neither absolutely nor relatively dominates $G_2 = \mathcal{J}_1$

In what follows we provide some of the characteristics that stable partitions must exhibit.

Proposition 1. In a stable partition $\mathcal{G}$

1. For each individual $i \in G_s$, $|G_s| > 1$, $s \in \{1, 2\}$, it must hold that

   (i) $A_{i,G_s} > A_{i,G_{s'} \cup \{i\}} \Rightarrow e_{i,G_s} + e_{i,G_{s'} \cup \{i\}} > \dfrac{2\beta}{\alpha}$, $s' \in \{1, 2\}$, $s' \neq s$,

   or

   (ii) $A_{i,G_s} < A_{i,G_{s'} \cup \{i\}} \Rightarrow e_{i,G_s} + e_{i,G_{s'} \cup \{i\}} < \dfrac{2\beta}{\alpha}$, $s' \in \{1, 2\}$, $s' \neq s$.

2. If individual $i \in G_s$, $|G_s| > 1$ is such that $b_i \geq \underline{b}(\alpha, \beta) = \beta/\alpha(1 + \beta)$ then it must hold that $A_{i,G_s} > A_{i,G_{s'} \cup \{i\}}$, $s, s' \in \{1, 2\}$, $s' \neq s$.

3. If $b_i \geq \underline{b}(\alpha, \beta)$ for each $i \in \mathcal{N}$ then all the individuals can be organized in one group, and the alternative group is therefore empty.

Point 1 describes how the benefits of belonging to a particular group depend on the extent to which an individual can take advantage of others' average performance. Such an individual derives the highest utility when she is part of a group with the highest average performance (condition 1.(i)) if she is able to exert sufficiently high efforts so that, overall, the prospects of global quality and local standing benefit her. An instance in which this happens is when such an individual is productive enough, in particular when her productivity is above the threshold $\underline{b}(\alpha, \beta)$ described in point 2, as then she is able to exert a level of effort higher than β/α in any group.[19]

The ratio β/α has a very intuitive meaning, it measures the importance of local standing relative to global quality. More specifically, the higher α the higher the extent to which complementarities can be exploited, meaning that even with a smaller level of effort it is still beneficial for an individual to belong to a group in which others' average performance is high. The contrary happens with β, the larger its value the higher the effort should be for an individual to be able to overcome disadvantageous local comparisons in a group where

[19] That can be assessed by plugging a value $b_i > \underline{b}(\alpha, \beta)$ in $e_{i,G}(e_{-i,G}) = b_i(1 + \beta) + \alpha A_{i,G}$.

others' average performance is high. The interpretation for condition 1.(ii) is analogous in the natural opposite direction.

Point 2 simply tells that if an individual is sufficiently productive no group should be available to her in which she experiences a higher average performance than in her own group.

The observations in the previous points give rise to the conclusions in the following Corollary 1.

Corollary 1. Let $\mathcal{G}$ be a partition that contains non-singleton groups G_s and $G_{s'}$, $s \neq s'$. Then

1. $b_i \geq \underline{b}(\alpha, \beta)$ for each $i \in G_s$ such that $A_{i,G_s} > A_{i,G_{s'} \cup \{i\}}$ is a sufficient condition for 1.(i) of Proposition 1 to hold.
2. $b_i < \underline{b}(\alpha, \beta)$ for each $i \in G_s$ such that $A_{i,G_s} < A_{i,G_{s'} \cup \{i\}}$ is a necessary condition for 1.(ii) of Proposition 1 to hold.

The statements made in Corollary 1 are useful because they inform about the possibilities of stability in terms of the primitives of the model. Also, they are useful in situations in which we know how, from the point of view of an individual i, the average performance in the group she is evaluating whether to move relates to the one in her current group. In this case, it is enough to look at how productive an individual is to assess, up to a certain extent, the absence of incentives to switch groups.[20]

Point 3 states that individuals can be always organized in one group whenever they are sufficiently productive.

4.2.1 Specific classes of partitions

Prominent partitions are those in which groups are consecutive, and hence there is grouping by productivity, and those in which (some of the) groups are non-consecutive, thus exhibiting a mixing of productivities. Guided by the prescriptions in Proposition 1 we study when such partitions can be stabilized.

The following Lemma 2 provides the conditions for stability of a partition in which there is grouping by productivity and thus a group, say G_1, (A)bsolutely (D)ominates the other, G_2. We thus refer to such a partition as an AD-partition and emphasize that the individuals in G_1 face a lower average performance if they move to G_2 whereas the individuals in G_2 face a higher average performance if they move to G_1.

[20] That is the case for the partitions studied in the subsequent Lemma 2 and also for the ones in Lemma 3, for some of the individuals.

Lemma 2. Let $b_i < \underline{b}(\alpha,\beta)$ for some $i \in \mathcal{N}$. Then an AD-partition $\mathcal{G} = \{G_1, G_2\}$ where $|G_1|, |G_2| > 1$ is stable if and only if

1. $e_{i,G_1} + e_{i,G_2\cup\{i\}} \geq \dfrac{2\beta}{\alpha}$ for each $i \in G_1$,

   and

2. $e_{j,G_2} + e_{j,G_1\cup\{j\}} \leq \dfrac{2\beta}{\alpha}$ for each $j \in G_2$.

Condition 1 of Lemma 2 tells that no individual in group G_1, consisting of the individuals with the highest productivities, has incentives to move to G_2, the group formed by the individuals with the smallest productivities, when she is productive enough so that she can take advantage of a high average performance. An analogous interpretation follows for condition 2, describing the incentives of an individual in G_2. Such an individual does not have incentives to move to G_1, when her productivity is small enough so that she cannot take advantage of a high average performance. Condition 2 also implies that the productivity of any individual in G_2 must be sufficiently low, in particular, it must hold that $\max_{j\in G_2} b_j < \underline{b}(\alpha,\beta)$, and thus such a restriction on the magnitude of private productivities defines an upper bound on the cardinality of G_2 and a lower bound on the cardinality of G_1.

Example 2. Consider a population $\mathcal{N} = \{1,2,3,4\}$ and let $G_1 = \{1,2\}$, $G_2 = \{3,4\}$. Let also $(b_1,b_2,b_3,b_4) = (0.6, 0.5, 0.05, 0.025)$ and $\alpha = \beta = 1$ so that $2\beta/\alpha = 2$. We analyze individual incentives to remain in their prescribed groups by considering first individuals in G_2. For individual 3 direct computations lead to that $e_{3,G_1\cup\{3\}} = 1.65$ and $e_{3,G_2} = 0.11$, therefore the left-hand side of condition 2 in Lemma 2 equals 1.76, meaning that she does not have incentives to move to G_1. It can be shown that this is also the case for individual 4. Note that individuals in G_1 do not have incentives to abandon their group since they are productive enough according to the threshold $\underline{b}(\alpha,\beta) = 0.5$ specified in point 1 of Corollary 1.

In line with the anecdotal evidence discussed in the introductory Section 1 we would be also interested in stabilizing partitions in which there is a mixing of productivities. With this aim in mind the following Example 3 describes a class of partitions in which one group (R)elatively (D)ominates the other and we refer to any partition in this class as a RD-partition.

Example 3. **A class of RD-partitions.** Let $N \geq 5$ be an odd integer and partition $\mathcal{G} = \{G_1, G_2\}$ be such that G_1 and G_2 are non-consecutive with $|G_1| = |G_2| + 1$. Specifically

1. Let G_1 be the union of two intervals, $G_1 = \mathcal{I}_1 \cup \mathcal{I}_2$, such that

(a) $|\mathcal{I}_1| = |\mathcal{I}_2|$ whenever $2^{-1}(N-1)+1$ is even,

and

(b) $|\mathcal{I}_1| = |\mathcal{I}_2| + 1$ otherwise.

2. Let G_2 be the union of two intervals, $G_2 = \mathcal{J}_1 \cup \mathcal{J}_2$, such that $|\mathcal{J}_1| = 1$ and $|\mathcal{J}_2| = 2^{-1}(N-1) - 1$.

We find the case $N = 5$ useful to understand how individual's incentives should be shaped in order to sustain partitions in this class as stable. Then, in such a case we have that $G_1 = \mathcal{I}_1 \cup \mathcal{I}_2 = \{1,2\} \cup \{4\}$ and $G_2 = \mathcal{J}_1 \cup \mathcal{J}_2 = \{3\} \cup \{5\}$.

1. First, note that each individual $i \in G_1$ would face a smaller average performance in $G_2 \cup \{i\}$ as basically such an individual i that moves from G_1 to G_2 is giving up a peer with higher productivity and choosing a peer with smaller productivity. For instance, individual 1 gives up individuals 2 and 4 and chooses 3 and 5, respectively. Then, in line with the discussion of the results in Proposition 1, individual 1 would not have incentives to move to G_2 when her efforts are high enough so that she takes advantage of a higher average performance in G_1.

2. Second, note that the contrary happens to individual $3 \in \mathcal{J}_1 \in G_2$ for whom others' average performance is higher in $G_1 \cup \{3\}$, as all the individuals currently in G_1 are more productive than individual 5. Then, in line with the discussion of the results in Proposition 1, individual 3 would not have incentives to move to G_1 when her efforts are low enough for her to be able to take advantage of a higher average performance in $G_1 \cup \{3\}$.

3. Third, individual $5 \in \mathcal{J}_2 \in G_2$ may either face higher or lower average performance if she moves to G_1, and that very much depends on the parameter values. In the example above individual $5's$ peers in $G_1 \cup \{5\}$ are 1 and 2, who are more productive than 3, but also 4, who is less productive and 3.

Relying on these three insights the following result provides the conditions under which RD-partitions in this class emerge as stable.

Lemma 3. Consider the class of RD-partitions in Example 3 and let $b_i \leq \underline{b}(\alpha, \beta)$ for each $i \notin \mathcal{I}_1$. A RD-partition in this class is stable if and only if

1. $e_{i,G_1} + e_{i,G_2 \cup \{i\}} \geq \dfrac{2\beta}{\alpha}$ for each $i \in G_1$,

2. $e_{j,G_2} + e_{j,G_1 \cup \{j\}} \leq \dfrac{2\beta}{\alpha}$ for $j \in \mathcal{J}_1$

and

3. For each $k \in \mathcal{J}_2$

   (i) if $A_{k,G_2} > A_{k,G_1 \cup \{k\}} \Rightarrow e_{k,G_2} + e_{k,G_1 \cup \{k\}} \geq \dfrac{2\beta}{\alpha}$,

   or

   (ii) if $A_{k,G_2} < A_{k,G_1 \cup \{k\}} \Rightarrow e_{k,G_2} + e_{k,G_1 \cup \{k\}} \leq \dfrac{2\beta}{\alpha}$.

Example 4. Consider the case $N = 5$ above with $G_1 = \mathcal{I}_1 \cup \mathcal{I}_2 = \{1, 2\} \cup \{4\}$ and $G_2 = \mathcal{J}_1 \cup \mathcal{J}_2 = \{3\} \cup \{5\}$. Let $\alpha = \beta = 1$, so that $2\beta/\alpha = 2$, and $(b_1, b_2, b_3, b_4, b_5) = (0.6, 0.5, 0.27, 0.26, 0.25)$. It then directly follows that individuals 1 and 2 do not have incentives to move to G_2 since again they are productive enough according to the threshold $\underline{b}(\alpha, \beta) = 0.5$. For individual 4 we have that $e_{4,G_1} = 1.48 + e_{4,G_2 \cup \{4\}} = 0.68 > 2$ implying that she does not have incentives to move to G_2. Finally, regarding individuals in G_2, note that individual $3 \in \mathcal{J}_1$ faces a higher average performance if she moves to G_1 but since $e_{3,G_2} = 0.718 + e_{3,G_1 \cup \{3\}} = 1.24 < 2$ she does not have incentives to do so. For individual 5, it is the case that $A_{5,G_2} = 0.194 < A_{5,G_1 \cup \{5\}} = 0.713$ and $e_{5,G_2} = 0.69 + e_{5,G_1 \cup \{5\}} = 1.21 < 2$, so that the condition in point 3.(ii) holds for her, which implies that she does not have incentives to move to G_1. We thus conclude that the exemplified RD-partition is stable.

Note that in any stable partition in this class, individual $j \in \mathcal{J}_1 \in G_2$ (individual 3 in Example 4) prefers to be a big fish (the highest productivity individual) in her group (the small pond, which is composed also by individual 5 in Example 4) than a relatively smaller fish in $G_1 \cup \{j\}$, that is, the individual with productivity in position $|\mathcal{I}_1| + 1$ out of $|G_1| + 1$ individuals (the third individual in $G_1 \cup \{3\} = \{1, 2, 3, 4\}$). The contrary happens to the most productive individual $i \in \mathcal{I}_2 \in G_1$ (individual 4 in the example), who prefers to be a relatively small fish in her high-quality group (the individual with productivity in position $|\mathcal{I}_1| + 1$ in her big pond G_1) than to be a relatively bigger fish in $G_2 \cup \{i\}$, that is, the individual with productivity in the second position out of $|G_2| + 1$ individuals. This latter effect is more salient the less productive the individual in $\mathcal{I}_2$ we consider.

To close this section, we would like to briefly discuss about the existence of stable partitions (in pure strategies) in our model by pointing out how in the extreme case in which local standing becomes unimportant, $\beta \to 0$, or group quality becomes increasingly important, $\alpha \to \infty$, then the set of primitives $(b_i)_{i \in \mathcal{N}}$ under which the partition consisting of just one group involving all the individuals becomes bigger (in the inclusion sense).[21]

The impossibility of guaranteeing the existence of stable partitions in pure strategies for any set of parameters is (in part) due to the free mobility of individuals among groups, which

[21] Such a partition is stable under the requirements on productivities described in point 3 of Proposition 1.

may be seen as a rather demanding assumption. We then may consider that stability relies on less restrictive conditions and conceive a partition as stable if (i) when an individual is willing to move from her group, (ii) a given number of individuals in the group she pretends to move veto her adhesion. That stability notion, defined below, is very much in the spirit of Watts (2007).

Definition 6. A partition is stable whenever for each individual $i \in G_s$, $s, s' \in \{1,2\}$, $s' \neq s$, $u_i(e_{i,G_{s'}\cup\{i\}}, e_{-i,G_{s'}\cup\{i\}}) > u_i(e_{i,G_s}, e_{-i,G_s}) \Rightarrow u_j(e_{j,G_{s'}\cup\{i\}}, e_{-j,G_{s'}\cup\{i\}}) < u_j(e_{j,G_{s'}}, e_{-j,G_{s'}})$ for at least a number $\lceil |G_{s'}|/2 \rceil$ of individuals $j \in G_{s'}$.

We relegate to Subsection 7.4 the discussion of how AD-partitions and the class of RD-partitions proposed in the main body can be sustained as stable in this case.

4.3 Increasing the importance of group quality ($\uparrow \alpha$), or of local standing ($\uparrow \beta$)

An interesting question is how stable partitions are shaped when group quality or local standing concerns are of increasing importance. Given the potential multiplicity of equilibria present in our model, we analyze changes in parameters α or β when we depart for a particular partition that is initially assumed to be stable.

The comparative statics exercise is not obvious since effort choices are endogenously determined and average performance enters into the individuals' utility function (see Eq. (2)) through two different channels, namely, group quality and local standing. As an illustration, consider that group quality concerns increase ($\uparrow \alpha$) so that equilibrium efforts increase as well (recall the efforts' best reply), then such an increase in turn boosts group quality and consequently individuals' local standing might be deteriorated. All these effects are quite sensitive to the values of the private productivities.

Despite of the difficulties highlighted above we are able to predict the direction in which a stable AD-partition reacts to changes in parameters, in specific scenarios.

Observation. Consider that for the primitives $(b_i)_{i\in\mathcal{N}}$, α, and β an AD-partition $\mathcal{G} = \{G_1, G_2\}$ is stable, then

1. Let α increase to $\alpha' > \alpha$ such that for the most productive individual $i \in G_2$ it holds that
$$\alpha' \geq \frac{\beta}{b_i(1+\beta)} > \alpha. \tag{3}$$
In this case, $\mathcal{G}$ ceases to be stable and, departing from it, any stable partition is an AD-partition $\mathcal{G}' = \{G'_1, G'_2\}$ such that $|G'_1| > |G_1|$ and $|G'_2| < |G_2|$.

2. Let β increase to $\beta' > \beta$ and $b_i \geq \beta'/\alpha(1+\beta')$ for each $i \in G_1$. In this case, $\mathcal{G}$ may cease to be stable. In such a scenario, departing from $\mathcal{G}$ any stable partition is an AD-partition $\mathcal{G}' = \{G'_1, G'_2\}$ such that $|G'_1| > |G_1|$ and $|G'_2| < |G_2|$.

Point 1 of this observation illustrates the case in which group quality concerns become stronger. In particular, we consider an increase in α that precludes condition (ii) in Lemma 2 to hold for some individuals. Such a change incentivizes, at least, the most productive individual in $i \in G_2$ to move to G_1. Notice that no individual $j \in G_1 \cup \{i\}$ has incentives to move to $G_2 \setminus \{i\}$ as for her the condition in Eq. (3) automatically holds since she is, by definition, more productive than any individual in G_2. That means that condition 1 in Lemma 2 necessarily holds for individuals in G_1.

If after the departure of the most productive individual $i \in G_2$, who moves to move to G_1, condition 2 of Lemma 2 holds for each individual $j \in G_2 \setminus \{i\}$, we have already reached a new stable AD-Partition, otherwise, there is an individual $j \in G_2$ who would like to move to G_1 and so on.

Thus, in conclusion, sufficiently strong concerns for group quality will cause the movement of low-productivity individuals to the group in which average performance is higher.[22]

Point 2 of the observation above illustrates the case in which local standing concerns become stronger, that is, β increases. Suppose also that α is already sufficiently high so that the right-hand side of condition 2 of Lemma 2 tends to be small. In this case, an increase in β boosts efforts sufficiently so that some individuals may be now willing to move to the group in which they face the highest average performance, that is, condition 2 of Lemma 2 may end up be violated for them. A more technical argument emerges by taking a look at such a condition: its left-hand side reveals that the larger α the larger the effect that an increase in β has on others' average performance, and thus the larger the effect on effort choices, in contrast, in the right-hand side the larger α the smaller the effect of an increase in β.[23]

5 Social welfare

We analyze here the relevant question of how efforts and partitions should be designed in order to maximize social welfare, and for this purpose we consider two different measures of

[22] An analogous analysis could be also applied to a situation in which instead of having common group quality concerns, individuals differ in the importance they grant to such an indicator. Thus, we move from having common α to a setup in which $\alpha_i \neq \alpha_j$ for some pairs $i \neq j$ while we keep β constant. For instance, suppose that for common α the AD-partition $\mathcal{G} = \{G_1, G_2\}$ is stable. Then, if for some individuals $i \in G_2$ we have that concerns for group quality increase to $\alpha_i > \alpha$, such individuals may be already willing to move to G_1.

[23] Formally for each $i \in G$: $\partial e_{i,G}/\partial\alpha\partial\beta = \partial A_{i,G}/\partial\alpha\partial\beta > 0$ and $\partial^2(\beta/\alpha)\partial\alpha\partial\beta = -1/\alpha^2 < 0$.

social welfare, one of them being the sum of individual utilities and the other, the sum of exerted efforts.

5.1 Social welfare as the sum of individual utilities

Let social welfare be defined as the sum of individual utilities, that is

$$\mathcal{W} \equiv \sum_{G\in\mathcal{G}} \sum_{i\in G\in\mathcal{G}} u_i(e_{i,G}, e_{-i,G}).$$

We say that an outcome pair composed of a partition $\mathcal{G}$ and individual efforts $e_{\mathcal{G}} \equiv (e_{i,G\in\mathcal{G}})_{i=1,\dots,N} \in \mathbb{R}^N_+$ is socially optimal, or efficient, when it maximizes $\mathcal{W}$. Thus, we are interested in characterizing the efforts that maximize the sum of utilities within a given group G, namely the efficient efforts. Let $e^E_{i,G}$ denote the efficient effort of individual $i \in G$ and e^E_G be the vector of efficient efforts of all the individuals in G. In the same vein, let $A^E_{i,G}$ be others' average performance, when all the individuals in G exert efficient efforts.

Proposition 2. Consider a group $G \in \mathcal{G}$. Then, for each $i \in G$ the efficient effort satisfies

$$e^E_{i,G} = b_i \left[1 + \alpha \bar{e}^E_{i,G}\right] + \alpha A^E_{i,G}, \tag{4}$$

where $\bar{e}^E_{i,G} = [|G| - 1]^{-1} \sum_{j\neq i} e^E_{j,G}$. Given efficient efforts, social welfare, amounts (up to a constant) to the sum of individuals' private products, specifically

$$\sum_{G\in\mathcal{G}} \sum_{i\in G\in\mathcal{G}} u_i(e^E_{i,G}, e^E_{-i,G}) = 2^{-1} \sum_{G\in\mathcal{G}} \sum_{i\in G\in\mathcal{G}} b_i e^E_{i,G}.$$

It is direct to observe that efficient efforts do not incorporate concerns for local standing, that recall, are modulated by β, as in the aggregate the utility gains of individuals who stand above others' average performance offset the utility losses of individuals who stand below others' average performance, thus the local standing effect cancels out. Further, as the effort exerted by an individual i (positively) affects all others' utility via group quality, efficiency requires the inclusion of such a positive externality, an effect that is captured through the term $\alpha\bar{e}^E_{i,G}$ in Eq. (4).[24]

Efficient efforts, described by Eq. (4), differ from Nash equilibrium efforts, described by the best reply $e_{i,G}, (e_{-i,,G}) = b_i(1 + \beta) + \alpha A_{i,G}$. Only when all the individuals exert the same effort level, $e_{i,G} = \beta/\alpha$ for each $i \in G$, then efficient and Nash equilibrium efforts coincide but

[24] Analogously to the case of Nash equilibrium efforts, there are requirements for efficient efforts to exist. We refer the interested reader to Subsection 7.5 in the Appendix. Also, note that when G is only composed of one individual i, then the efficient effort is $e^E_{i,G} = b_i$.

it actually follows that this symmetric proposal cannot be either a solution of the system of best replies or a solution to the system describing efficient efforts, thus the set of efficient and Nash efforts is empty. This result, together with some additional aspects, is stated below.

LEMMA 4. No effort profile can be simultaneously efficient and a Nash equilibrium. Specifically, $e_G^E - e_G$ has at least one entry different from zero. Further, for each individual i, the difference $e_{i,G}^E - e_{i,G_s}$

1. Increases as α increases.
2. Decreases as β increases.
3. Increases as b_j increases, for any $j \in G$ (possibly $j = i$).

The discrepancy between efficient efforts and Nash equilibrium efforts depends on the importance of local standing, modulated by β, and group quality, modulated by α. If group quality is of great importance efficient efforts tend to be the highest because in the aggregate higher efforts by individuals positively affect all others via high average performance. If on the contrary local standing is of great importance Nash equilibrium efforts increase whereas efficient efforts do not. Finally, the more productive individuals are, the more positive the difference between efficient and Nash equilibrium efforts, because intuitively efficiency requires that the effect of an increase in individuals' productivity permeates all others' efforts via group quality. Finally, note that for some individuals efficient efforts may be higher than Nash equilibrium efforts while for others this relation may be reversed.[25]

EXAMPLE 5. As a follow-up of Example 1, recall that Nash equilibrium efforts are $e_{1,G} = 1.9$ and $e_{2,G} = 1.56$ whereas for the same parameter values efficient efforts amount to

$$e_{1,G}^E = \frac{\alpha(b_1 + b_2)b_2 + b_1}{1 - \alpha^2(b_1 + b_2)^2} = \frac{1.04}{0.64} = 1.62, \qquad e_{2,G}^E = \frac{\alpha(b_1 + b_2)b_1 + b_2}{1 - \alpha^2(b_1 + b_2)^2} = \frac{0.88}{0.64} = 1.37.$$

For both individuals, efficient efforts are below Nash equilibrium efforts when $\alpha = 0.5$ but if we consider a higher value $\alpha' = 0.57$, efficient efforts are above Nash equilibrium efforts, in particular, $e_{1,G} = 1.98 < e_{1,G}^E = 2.01$ and $e_{2,G} = 1.7 < e_{2,G}^E = 1.77$.[26]

[25] As a related aspect, see the discussion in Subsection 7.5 for an analysis of how to restore efficient efforts.

[26] For slightly smaller α' the efficient effort of individual 1 is below her Nash equilibrium effort and the contrary happens to individual 2. Finally, as an illustration of point 3 of Lemma 4, when the productivity of individual 2 increases from 0.4 to 0.7, efficient efforts become higher than Nash equilibrium efforts.

Finally, it is important to emphasize that contrary to Nash equilibrium efforts, efficient efforts are such that more productive individuals exert higher effort (and thus also generate higher private product).

We now study efficient partitions keeping in mind that in order to maximize social welfare we require that individuals exert efficient efforts. It is the case that in efficient efforts, individual productivities complement each other, more specifically, the reaction of an individual's effort to an increase in the productivity of another individual is stronger the higher the individual's own productivity, a prediction consistent with the findings by Ding and Lehrer (2007) that high-ability students benefit more from having higher-achieving schoolmates than students with lower ability do. This type of complementarity in productivities naturally leads to that efficient partitions must avoid productivity mixing and promote the grouping of individuals according to their productivities, thus only AD-partitions can be efficient. That is the content of the following result.

PROPOSITION 3. Partitions that maximize the sum of individual utilities consist of consecutive groups. Thus, if a group is a singleton, it must be composed of the least productive individual.

The specific architecture of the consecutive groups in a AD-partition is however sensitive to the primitives of the model. We exemplify the case $N = 4$ via simulations to illustrate which partitions consisting of consecutive groups are efficient. We find that partition $G_1 = \{1, 2\}$, $G_2 = \{(3, 4\}$ is efficient with a disproportionately high average frequency that lies in the range [84%, 92%] and that tends to increase with α, followed by partition $G'_1 = \{1, 2, 3\}$, $G'_2 = \{4\}$ which is efficient with an average frequency that lies, respectively, in the range [16%, 8%]. That suggests that complementarities in productivities are better exploited when the two most productive individuals form a group. There is however run for specific productivity values to play some role.[27]

Regarding the overall welfare effects of changes in α, β and productivities, note first that social welfare increases as α does regardless of whether, upon such an increase, the efficient partition consists of different groups than those that arise before the change. The reason is that the sum of individuals' private products that each partition induces increases with α and thus, the potential new efficient partition necessarily yields higher social welfare than the efficient one before the change. In the simplest case of two individuals, the efficient

[27] We run 1000 iterations each of which contains 1000 random draws of productivity values. The results hold for the all values of $\alpha \in \{0.25, 0.35, 0.45, 0.55\}$ considered, and when productivities are drawn from a Beta distribution $B(p, q)$ with $p = q = 1$, the high variance case, and two cases with smaller variance: (i) $p = 2, q = 1$ in which higher productivities are relatively more frequent, and vice versa for (ii) $p = 1, q = 2$. The code for the simulations and the resulting data appear in the supplementary material.

partition is the one in which these individuals form a group G because, intuitively, they both exert greater effort if they are grouped together than if they are isolated. In this case, social welfare is $2^{-1}[b_1 e^E_{1,G} + b_2 e^E_{2,G}]$ and since efficient efforts are increasing in α, so is this measure. Second, since efficient efforts are not affected by local concerns, social welfare does not vary with β. Finally, as efficient efforts are increasing in productivities, that is also the case for social welfare, again regardless of which one is the efficient partition after such an increase.

5.1.1 *The stability of efficient partitions*

A final interesting question is whether such efficient partitions can also be stable. Recall that for a partition to be efficient individuals must be exerting efficient efforts, thus to approach the assessment of stability in this context we assume that any individual in a given group evaluates whether she benefits from moving to the alternative group under the case in which she, and all the individuals in such an alternative group, exert efficient efforts.

To set the result, for $i \in G_s$ let $\overline{e}^E_{i,G_s \& G_{s'} \cup \{i\}} \equiv (\overline{e}^E_{i,G_s}, \overline{e}^E_{i,G_{s'} \cup \{i\}})$ and analogously for $e^E_{i,G_s \& G_{s'} \cup \{i\}}$, for $s' \neq s$.

Lemma 5. Consider that an AD-partition $\mathcal{G} = \{G_1, G_2\}$, where $|G_1|, |G_2| > 1$ is efficient. Then, such a partition is also stable according to Definition 2 if and only if

1. For each $i \in G_1$

$$e^E_{i,G_1} + e^E_{i,G_2 \cup \{i\}} + 2b_i \theta(\overline{e}^E_{i,G_1 \& G_2 \cup \{i\}}, e^E_{i,G_1 \& G_2 \cup \{i\}}) > 2\beta \left[\frac{1}{\alpha} - \frac{b_i(\overline{e}^E_{i,G_1} - \overline{e}^E_{i,G_2 \cup \{i\}})}{e^E_{i,G_1} - e^E_{i,G_2 \cup \{i\}}} \right];$$

2. For each $j \in G_2$

$$e^E_{j,G_2} + e^E_{j,G_1 \cup \{j\}} + 2b_j \theta(\overline{e}^E_{j,G_2 \& G_1 \cup \{j\}}, e^E_{j,G_2 \& G_1 \cup \{j\}}) < 2\beta \left[\frac{1}{\alpha} - \frac{b_j(\overline{e}^E_{j,G_1 \cup \{j\}} - \overline{e}^E_{j,G_2})}{e^E_{j,G_1 \cup \{j\}} - e^E_{j,G_2}} \right],$$

where

$$\theta(\overline{e}^E_{i,G_s \& G_{s'} \cup \{i\}}, e^E_{i,G_s \& G_{s'} \cup \{i\}}) = \frac{\alpha |\overline{e}^E_{i,G_s} e^E_{i,G_s} - \overline{e}^E_{i,G_{s'} \cup \{i\}} e^E_{i,G_{s'} \cup \{i\}}| - \beta |e^E_{i,G_s} - e^E_{i,G_{s'} \cup \{i\}}|}{|e^E_{i,G_s} - e^E_{i,G_{s'} \cup \{i\}}|}.$$

The conditions for stability in Lemma 5 contain additional elements than the parallel conditions in Lemma 2 and the reasons are that we now consider efficient efforts and, in contrast to Nash equilibrium efforts, such efficient efforts incorporate the positive externality that individuals impose on their peers via group quality.

From the point of view of an individual who evaluates whether to join an alternative group, the right hand side of the conditions above informs about the difference between others' average performance in her current group and in the alternative group times β, and thus reflects how local standing might be affected.[28] That expression is positive and smaller than $2\beta/\alpha$, which recall is the right hand side of the conditions in Lemma 2. The left hand side of the aforementioned conditions is, briefly, a re-expression of the sum of the private benefits that accrue to individual i when she exerts the efficient effort in her current and her alternative group, plus the between groups difference in social benefits due to group quality concerns.[29]

The mechanisms for stability are thus analogous to the ones already posed in Lemma 2, namely, an individual $i \in G_1$ would not have incentives to join G_2, in which individuals are of relatively smaller productivity than the ones of $i's$ current group, when, briefly, the improvement in local standing she may experience in G_2 is sufficiently small compared to the decrease in group quality when she moves to such a group. An analogous interpretation (in the natural opposite direction) follows for individuals in G_2.

Further, an implication of the comparative statics results on the difference between efficient and Nash equilibrium efforts in Lemma 4 is that for sufficiently small β, efficient efforts may overcome Nash efforts. When that is the case and θ takes positive values, the condition in point 1 would be less stringent for individuals in G_1 whereas the condition in point 2 would be more stringent for individuals in G_2, than the parallel conditions in Lemma 2.

For the aforementioned case $N = 4$, partition $G_1 = \{1, 2\}$, $G_2 = \{(3, 4\}$ is efficient and stable, according to Definition 2, with an average frequency that lies in the range $[25\%, 40\%]$ for $B(1, 1)$, $[42\%, 50\%]$ for $B(2, 1)$ and $[5\%, 32\%]$ for $B(1, 2)$. For partition $G'_1 = \{1, 2, 3\}$, $G'_2 = \{4\}$ average frequencies lie in $[1\%, 2\%]$ for $B(1, 1)$, $[2.6\%, 4\%]$ for $B(2, 1)$ and $[0.25\%, 1\%]$ for $B(1, 2)$.[30]

[28] Using efficient efforts in Eq. (4) we obtain that the right hand side of the condition in point 1 amounts to $2\beta(A_{i,G_1} - A_{i,G_2\cup\{i\}})/(e^E_{i,G_1} - e^E_{i,G_2\cup\{i\}})$. An analogous expression follows for the condition in point 2.

[29] Note that θ emerges from considering that the utility of an individual $i \in G$, who exerts the efficient effort, amounts to $2^{-1}(e^E_{i,G})^2 - \beta A^E_{i,G} + b_i e^E_{i,G}(\beta - \alpha\bar{e}^E_{i,G})$. Note also that θ can be interpreted as a measure of how much efficient efforts differ from Nash equilibrium efforts. More specifically, it can be shown that if from the point of view of individual i, others' average effort equals β/α in any group she belongs to, this expression cancels out. Also, the right hand side of the conditions in Lemma 5 would reduce to $2\beta/\alpha$, as in the conditions of Lemma 2.

[30] For $B(1, 2)$ frequencies of 5% and 0.25% are small compared to the lower bounds in the remaining distributions, but these values appear only for $\alpha = \beta = 0.25$. The reason for these outlier values is that small productivity values make it hard for individuals to be willing to remain in G_1, in particular, the left-hand side of the condition in point 1 of Lemma 5 is really small with respect to $2\beta/\alpha$. For higher values of α these average frequencies in fact exhibit a jump and are relatively closer to the ones of the remaining distributions. The reason why for distribution $B(2, 1)$, which favors high productivity values, stability may not be compromised, is the order of magnitude of the ratio $2\beta/\alpha$ with respect to productivity values and the

The conditions for efficient partitions to be stable according to Definition 6 are analogous to the ones posed in Lemma 5 and have also analogous interpretations. Such conditions would simply state that when an individual $i \in G_s$ has incentives to move to $G_{s'}$ then at least a number $\lceil G_{s'}/2 \rceil$ of individuals in $G_{s'}$ would veto her movement. Intuitively, an individual $i \in G_{s'}$ would veto the movement of an individual of say, smaller productivity than the individuals currently in $G_{s'}$, whenever, briefly, for such an individual i this change in group composition causes a decrease in group quality that overcomes the improvement in local standing. An analogous mechanism will be at work when an individual of higher productivity than the individuals currently in $G_{s'}$ pretends to join such a group. In this case, group quality would be improved, but some of the individuals may see their local standing deteriorated and thus veto the adhesion of such a new member.

Formally, individuals in G_1 would veto the movement of an individual $j \in G_2$ when for the majority of them a modified version of the condition in point 1 of Lemma 5 holds. In particular, for each $i \in G_1$ who vetoes $j's$ movement we require that

$$e^E_{i,G_1} + e^E_{i,G_1\cup\{j\}} + 2b_i\theta(\overline{e}^E_{i,G_1\&G_1\cup\{j\}}, e^E_{i,G_1\&G_1\cup\{j\}}) > 2\beta\left[\frac{1}{\alpha} - \frac{b_i(\overline{e}^E_{i,G_1} - \overline{e}^E_{i,G_1\cup\{j\}})}{e^E_{i,G_1} - e^E_{i,G_1\cup\{j\}}}\right].$$

Under such a condition individual i is worse off if j, who is an individual of relatively lower productivity, joins G_1. A direct implication that results from this expression is that when local standing is not important ($\beta \to 0$), local standing gains from admitting a member of lower productivity are hardly exploited, in particular the right hand side of the expression above becomes sufficiently small. Under small β, the left hand side also becomes bigger (θ increases). Thus, individual i is likely to veto the entry of a lower productivity member. An analogous modified version of the condition in point 2 should hold for an individual in G_2 who vetoes the movement of an individual $i \in G_1$.

Recall that under Definition 6 of stability the frequency with which efficient partitions are stable increases by definition, with respect to Definition 2, in fact for the case $N = 4$ efficient partitions are all stable under Definition 6 of stability.[31]

The stability of efficient partitions is also affected by changes in concerns for group quality, via α, or local standing, via β. To shed light on this issue it is perhaps useful to note that the utility that accrues to an individual $i \in G$ can be written, plugging efficient efforts described

fact that θ in the condition in point 2 of Lemma 5, which may take either sign, has a higher impact because of the more frequent high-productivity values.

[31] That is not always the case, if we diminish the importance of local standing by setting $\beta = 0.01$ and increase the importance of group quality by setting $\alpha = 1$ efficient partitions are not all stable.

by Eq. (4) in Eq. (2), as

$$2^{-1}e^E_{i,G}[b_i + \alpha[A^E_{i,G} - b_i\bar{e}^E_{i,G}]] + \beta[b_i e^E_{i,G} - A^E_{i,G}]. \tag{5}$$

As efficient efforts do not depend on β, we directly observe in Eq. (5) that if an individual's private product is below others' average performance, an increase in β hurts her and vice versa. In an efficient AD-partition, in which G_1 absolutely dominates G_2, the utility of the least productive individual in $i \in G_1$ decreases with β as she is below others' average performance in such a group, that is, $b_i e^E_{i,G_1} - A^E_{i,G_1} < 0$, but the utility she would experience in $G_2 \cup \{i\}$ would increase with β as she would be above others' average performance in such a group, that is, $b_i e^E_{i,G_2\cup\{i\}} - A^E_{i,G_2\cup\{i\}} > 0$. Thus, we expect that increasing values of β compromise the stability of efficient AD-partitions because of individuals that are below others' average performance in G_1 and above others' average performance if they move to G_2.

For the aforementioned case $N = 4$, the average frequency of efficient and stable (according to Definition 2) partitions monotonically decreases with increasing values of $\beta \in \{0.25, 0.35, 0.45, 0.55\}$, for each value of α and each of the three distributions of productivity values considered.

The effect of varying α seems more ambiguous, we observe in Eq. (5) that such a change would affect efforts, private products as well as (potentially) the distance between private product and others' average performance, whose importance also depends on β. From the discussion above, we emphasize that for relatively high values of β stability may be compromised, even if α increases.

For the case $N = 4$ described above, changes in α in fact yield more mixed results than those for the case in which β increases, but some patterns can still be summarized.

1. Partition $G_1 = \{1, 2\}$, $G_2 = \{3, 4\}$. For a relatively high $\beta = 0.55$ such a partition is efficient and stable with an average frequency that increases when α increases from 0.25 to 0.35 but that weakly decreases for higher values of α (for $B(1, 1)$ and $B(2, 1)$), suggesting also that an increase in α may have more impact if it happens at small values of such a parameter. For the aforementioned distributions and $\beta < 0.55$, such an average frequency shows an increasing tendency, with minor fluctuations, as α increases. Finally, for $B(1, 2)$ such an average frequency increases with α for all values of β.[32]

[32] When β is relatively smaller, the individuals i that may compromise stability the most are the most productive ones in G_2 whose utility may decrease with α in such a group, as for each of them $A^E_{i,G_2} - b_i\bar{e}^E_{i,G_2} < 0$, and whose utility may increase if they move to G_1, as they are the least productive individuals in such a group and thus, $A^E_{i,G_1\cup\{i\}} - b_i\bar{e}^E_{i,G_1\cup\{i\}} > 0$. The results however suggest that this effect is not that relevant, in particular for $B(1, 2)$.

2. Partition $G'_1 = \{1,2,3\}$, $G'_2 = \{4\}$. Such a partition is only efficient and stable for relatively small $\beta = 0.25$. Its average frequency shows a decreasing tendency for $B(2,1)$ and $B(1,1)$ and the reason may be that under such distributions high-productivity values are relatively (more) frequent. Thus, individual 4 tends to have a relatively high productivity and therefore (i) her exerted effort in $G'_1 \cup \{4\}$ would be high not only because of her (relatively high) productivity but also because of others' (relatively high) efforts and, additionally, (ii) as she is the least productive individual in such a group, $A^E_{4,G'_1\cup\{4\}} - b_4\bar{e}^E_{4,G'_1\cup\{4\}} > 0$ holds. Thus, in conclusion, she may have strong incentives to move to G'_1. Finally, for $B(1,2)$, which favors low-productivity values, such an average frequency weakly increases with α, which may suggest that a lowly productive individual 4 does not have incentives to move to G'_1.

5.2 Social welfare as the sum of individual efforts

We take here the approach of considering that there is a social planner who is concerned with maximizing the sum of efforts

$$\mathcal{W} \equiv \sum_{G\in\mathcal{G}} \sum_{i\in G\in\mathcal{G}} e_{i,G}.$$

That seems to be a natural objective, in order to improve performance professors are often concerned with motivating their students and achieve higher exerted efforts, and managers may also want to incentivize their employees in this sense.

To analyze this question we need to make a choice on how individuals are going to behave once they are grouped together, hence we consider that a planner can only design partitions (into two groups) and that he knows that the individuals within a group would play Nash equilibrium efforts. It turns out that the Nash equilibrium effort of each individual i in a given group is increasing in $i's$ productivity and the more productive i is the more sensitive she is to other's productivity (as in the case studied in Subsection 5.1), thus the efficient partitions in this case also consist of consecutive groups, as the following result states.

PROPOSITION 4. Partitions that maximize the sum of individual efforts consist of consecutive groups. Thus, if a group is a singleton, it must be composed of the least productive individual.

We summarize the findings regarding the conflict between efficiency and stability for AD-partitions in this case. If for some $i \in G_2$ it holds that $b_i \geq \underline{b}(\alpha,\beta)$, no efficient partition is stable under Definition 2 and efficient partitions are always stable under Definition 6. Note that in this case some of the individuals in G_2 are willing to move to G_1 and whereas under the former definition there is free mobility, under the latter definition individuals in G_1, who are sufficiently productive as well, will exert their veto power. In fact, if only individuals in G_1 are sufficiently productive according to the specified threshold, an efficient partition is stable according to Definition 6 because in the case that some individual in G_2 would like to move to G_1 they will exert their veto power. In more general terms, for an efficient partition to be stable according to Definition 2 conditions in Lemma 2 should be satisfied.

Finally, it is also direct to assess that overall welfare increases with α, β, and individual productivities because Nash equilibrium efforts are increasing in such primitives.

6 Conclusions

This paper studies a model in which individuals are concerned with the quality of their group and with their standing within such a group. In contrast with the majority of papers within this literature, both, group configurations and effort choices are endogenously determined within the model.

Our main aim is to shed light on the relationship between effort choices and group formation in environments other than firms. In this setting, we rationalize the emergence of group configurations resembling segregation but also a mixing of productivities, which is consistent with anecdotal evidence.

In our model, when social welfare is defined as the sum of individual utilities, equilibrium efforts are never efficient. In light of this result, we offer (Subsection 7.5) a discussion of the tax/subsidy scheme that a social planner would implement to recover efficiency. How the fact that individuals anticipate the introduction of tax and subsidies affects the formation of groups in the first stage is an interesting question that is left for further research.

Regarding group configurations, efficiency requires that groups are consecutive. We would like to emphasize how this result is due, of course, to the functional form of the individuals' utility but also to our choice of the social welfare function. In particular, we care about the sum of individual utilities or of individual efforts and we assume that all the individuals are equally important. In this case the complementary nature of individual productivities leads to the results stated above.

If alternatively we would consider that some, maybe the less productive individuals, are more important from the social point of view the result would probably change in favor of a mixing of productivities. More specifically, and in the context of the important debate on whether schools should introduce ability tracking or not, we would like to clearly state that we are not advocating that ability tracking is better than ability mixing. This is mainly an empirical question, very sensitive to the characteristics of the environment and therefore a model that incorporates other aspects more specific to the analysis of educational policies would probably be needed.

Regarding the choice of groups in the first stage of the game, an interesting venue of research would be to consider that individuals have an inherent preference for joining one of the two groups, apart from how productive are the peers that will ultimately form these groups. It might be that individuals identify with a given group, for instance a university or a research department, because of its (political) values and that this makes individuals more prone to choose a particular institution.

In relation to the local standing part of the utility specification in Eq. (2), an interesting venue of research entails considering that individuals are more affected by losses when they stand below others' average performance than by gains when they stand above others' average performance (Tversky and Kahneman, 1991).

To close this section, we would like to point out how Definition 6 of stability allows to further think in related scenarios in which individuals who hold veto power may be willing to prioritize either group quality or their local standing and are able to do so by influencing the group formation process. That perspective opens the possibility of interesting research venues.

7 Appendix

In this section, we discuss relevant extensions of the baseline model and also aspects related to the existence of stable partitions and efficient efforts (and how to restore them). We finally present the technical proofs.

7.1 Externalities between groups

To study the role of externalities between groups we follow the approach of considering that for each individual in a given group, the average performance of individuals in the alternative group may affect her positively (for instance, when for research departments it is beneficial that other departments' performance is high in order to establish work alliances) or negatively (it may be the case that for students in a given university, other universities' performance/position in a ranking negatively affects their chances of accessing to better jobs or getting grants).[33]

[33] Research studying the role of externalities in group formation includes Yi (1997), whose focus is on the impact of the sign of externalities on equilibrium outcomes under different rules for coalition formation, Konishi et al. (1997), who analyze group formation in games with network externalities or Pinto et al. (2015), who study the formation of societies under positive externalities (conformity games) and negative externalities

More formally, for an individual $i \in G_s$, $s \in \{1,2\}$, let $T_{G_{s'}} = \sum_{k \in G_{s'}} b_k e_{k,G_{s'}} / |G_{s'}|$ be the average performance of individuals in $G_{s'}$, $s' \in \{1,2\}$, $s' \neq s$. Thus, we consider an extended version of the baseline model which allows for the possibility that each individual $i \in G_s$ is affected by the average performance in group $G_{s'}$, so that in this case individual $i's$ utility is generally expressed as $v_i(e_{i,G_s}, e_{-i,G_s}, T_{G_{s'}})$. Below we define when this extended model exhibits positive or negative externalities.

Definition 7. **Positive externalities (PE).** The model exhibits positive externalities if for each $i \in G_s$, $v_i(e_{i,G_s}, e_{-i,G_s}, T_{G_{s'}}) \geq v_i(e_{i,G_s}, e_{-i,G_s}, \hat{T}_{G_{s'}})$ for $T_{G_{s'}} \geq \hat{T}_{G_{s'}}$, $s, s' \in \{1,2\}, s' \neq s$.

Definition 8. **Negative externalities (NE).** The model exhibits negative externalities if for each $i \in G_s$, $v_i(e_{i,G_s}, e_{-i,G_s}, T_{G_{s'}}) \leq v_i(e_{i,G_s}, e_{-i,G_s}, \hat{T}_{G_{s'}})$ for $T_{G_{s'}} \geq \hat{T}_{G_{s'}}$, $s, s' \in \{1,2\}, s' \neq s$.

With the purpose of offering preliminary results on the role of externalities between groups, we consider in particular that $v_i(e_{i,G_s}, e_{-i,G_s}, T_{G_{s'}}) = u_i(e_{i,G_s}, e_{-i,G_s}) m(T_{G_{s'}})$, $m \in \{f, g\}$, where $u_i(e_{i,G_s}, e_{-i,G_s})$ is described by Eq. (2) and we restrict the attention to the case in which it takes positive values. Further, m is a function that takes positive values and in the PE case $m = f$, where f is increasing in its argument, whereas in the NE case $m = g$, where g is decreasing in its argument.[34] This specification allows us to guarantee that the best reply of individual efforts is analogous to the one in the model without externalities between groups, so that we can compare both approaches in a smooth way. Still, this specification is flexible enough to accommodate the case in which externalities enter in an additive separable way (for instance when $m(x) = 1 + x/u_i(.,.)$) or in a multiplicative way. We present below a preliminary result.

Lemma 6. For each $i \in G_s$, $s \in \{1,2\}$ and $s' \in \{1,2\}$, $s' \neq s$, let $v_i(e_{i,G_s}, e_{-i,G_s}, T_{G_{s'}}) = u_i(e_{i,G_s}, e_{-i,G_s}) m(T_{G_{s'}})$. Then, with respect to the model without externalities between groups, the condition that guarantees that no individual $i \in G_s$ has incentives to move to $G_{s'}$

1. Is harder to be met if $T_{G_{s'}} < T_{G_s \setminus \{i\}}$ and there are positive externalities, or if $T_{G_{s'}} > T_{G_s \setminus \{i\}}$ and there are negative externalities.

(congestion games). Also, see Bloch (2005) for an analysis of the role of externalities with applications to industrial organization.

[34] As long as m takes positive values, the analysis of the case in which the utility function takes negative values is analogous to the one presented, with the difference that f should be decreasing in its argument and vice versa for g.

2. Is easier to be met if $T_{G_{s'}} < T_{G_s \setminus \{i\}}$ and there are negative externalities, or if $T_{G_{s'}} > T_{G_s \setminus \{i\}}$ and there are positive externalities.

Notice that when an individual $i \in G_s$ evaluates whether to move to $G_{s'}$ she takes into account that her alternative group would become $G_s \setminus \{i\}$ if she decides to do so, thus the relevant comparison for individual i is between $T_{G_{s'}}$ and $T_{G_s \setminus \{i\}}$. Thus, if for such an individual the average performance in the group she would abandon is in fact high enough she would have even more incentives to move when such an average performance benefits her (PE case) and less incentives to do so when such an average performance hurts her (NE case), than in the framework without externalities.[35]

An implication of the aforementioned results is that the sign of externalities affects the incentives of individuals in different ways and also that individuals in different groups assess externalities differently. For instance, an AD-partition that is stable in the model without externalities may fail to be stable in the PE case and the reason is that an individual $i \in G_1$ may now have incentives to move to G_2 because once she moves to such a group she faces an alternative group with high average performance (such group would be $G_1 \setminus \{i\}$ and note that average performance in such group is higher than in G_2). In the NE case such an AD-partition may also fail to be stable but now the reason is that an individual in $i \in G_2$ may have incentives to move to G_1 and hence face a smaller average performance in the alternative group, which now is $G_2 \setminus \{i\}$. Analogous mechanisms are at work for the case of RD-partitions depending on whether individuals face higher or lower average performance in an alternative group when they evaluate whether to move, and on the sign of externalities.

7.2 Incomplete information about the productivity of individuals

In some environments, it seems natural that individuals face uncertainty about others' attributes, for instance, the actual productivity of other students in the classroom or colleagues in a research department may not be known with certainty. We formalize this idea by considering that there is incomplete information about individuals' productivity. As we shall show, the results are (with their own particularities) smooth extensions of the ones in the main body.

Let $\mathcal{N}$ be a set with a population of N individuals. Each individual is labeled as $i \in \{1, 2, ..., N\}$ and characterized by an exogenous productivity parameter that defines her type. Nature assigns each individual a type and individuals learn their own type but not other

[35] As long as m takes positive values, the case in which the utility of an individual i in her own group and in the alternative group she may move to have opposite signs, the condition for stability either always hold in both, the model with externalities and the model without externalities or does not hold in either model.

individuals' types. Let $\mathcal{B}$ be a finite set of types and $B_i \subseteq \mathcal{B}$ be the set of types that individual $i \in \mathcal{N}$ can be assigned with positive probability. Let $b \in \mathcal{B}$ be a particular type profile for which we sometimes use the notation (b_i, b_{-i}) to separate between the type of an individual i and the types of the individuals different from i.

A pure strategy for an individual i is a mapping $s_i : B_i \to g_i \times e_i$, such that when individual i is assigned type b_i, $s_i(b_i)$ is a pair consisting of $g_i(b_i) \in \{G_1, G_2\}$ and $e_i(b_i)$. Note that $e_i(b_i)$ is itself a mapping from the set of possible profiles of group choices, each of them $g \equiv (g_1, g_2, ...g_N)$, and for each collection b_{-i} of others' types, to the effort exerted by the type b_i of individual i. Let finally S_i be the set of pure strategies available to individual i. Individuals' payoffs depend on the strategies and types of all individuals. Formally, let $S = \times S_i$ and $B = \times B_i$ and consider that for each $i \in \mathcal{N}$ there utility function $u_i : S \times B \to \mathbb{R}$ that takes the form in Eq. (2).

We assume that individuals have consistent beliefs with respect to the probability distribution of type profiles $p : \mathcal{B} \to \mathbb{R}_+$, according to which players are assigned their types, where $p(b)$ is the joint probability distribution of the type profile b, so that $\sum_{b \in \mathcal{B}} p(b) = 1$. Finally, let $p(b_{-i}|b_i)$ be the conditional probability that other individuals are assigned types b_{-i} when individual i is assigned type b_i.

The equilibrium notion is Perfect Bayesian equilibrium, which in this case reduces to ensure that for each profile of group choices g the efforts exerted by individual types constitute a (Bayesian) Nash equilibrium and that no individual type has incentives to deviate from its group.

Definition 9 and Definition 10 help to formalize these ideas. To do so, it is useful to simply consider that for a fixed profile of group choices g in which the type b_i of individual i chooses to belong to an arbitrary group G, $e_{i,G}(b_i)$ is the effort exerted by such a type in group G, for each particular collection of others' types b_{-i} who also choose G. Let also $e_{i,g}$ denote the collection of efforts exerted in g by all the types of individual i.

Definition 9. Fix a profile of group choices $g \equiv (g_1, g_2, ...g_N)$. Then, the effort choices $(e_{1,g}, e_{2,g}, ..., e_{N,g})$ constitute a Bayesian Nash equilibrium of the effort choice subgame whenever for each individual i and each of her types $b_i \in G$ it holds that

$$\sum_{b_{-i} \in \mathcal{B}_{-i}} u_i(e_{i,G}(b_i), e_{-i,G}(b_{-i}), b_i, b_{-i}) p(b_{-i}|b_i) \geq$$

$$\sum_{b_{-i} \in \mathcal{B}_{-i}} u_i(e'_{i,G}(b_i), e_{-i,G}(b_{-i}), b_i, b_{-i}) p(b_{-i}|b_i), \quad e'_{i,G}(b_i) \neq e_{i,G}(b_i) \quad \text{for} \quad \text{some} \quad b_{-i}.$$

Definition 10. A profile of group choices $g \equiv (g_1, g_2, ...g_N)$ is stable whenever for each individual i and each of her types $b_i \in G_s$, $s, s' \in \{1,2\}$ and $s' \neq s$ it holds that

$$\sum_{b_{-i}\in\mathcal{B}_{-i}} u_i(e_{i,G_s}(b_i), e_{-i,G_s}(b_{-i}), b_i, b_{-i})p(b_{-i}|b_i) \geq$$
$$\sum_{b_{-i}\in\mathcal{B}_{-i}} u_i(e_{i,G_{s'}\cup\{b_i\}}(b_i), e_{-i,G_{s'}\cup\{b_i\}}(b_{-i}), b_i, b_{-i})p(b_{-i}|b_i).$$

We highlight here the parallelisms with the results in the baseline model and comment on the particularities attached to this framework. We mainly illustrate the case of AD-partitions, and then emphasize that the case of RD-partitions relies on analogous intuitions.

1. *AD-profiles.* We say that a profile of group choices is an AD-profile when for each type profile b, one of the groups, say G_1, absolutely dominates the other, G_2.

For an AD-profile to be stable according to Definition 2 (the conditions being analogous to the ones of Lemma 2) it must be the case that for each individual i none of her types b_i has incentives to deviate from its group (recall that types in G_1 face a higher average performance in such group than if they move to G_2 and vice versa for types in G_2). Specifically, for each type $b_i \in G_1$ it must hold that

$$\sum_{b_{-i}\in B_{-i}} (A_{i,G_1}(b_i, b_{-i}) - A_{i,G_2\cup\{k\}}(b_i, b_{-i}))[2^{-1}\alpha(e_{i,G_1}(b_i) + e_{i,G_2\cup\{i\}}(b_i)) - \beta]p(b_{-i}|b_i) \geq 0,$$

and for each type $b_j \in G_2$ it must hold that

$$\sum_{b_{-j}\in B_{-j}} (A_{j,G_2}(b_j, b_{-j}) - A_{j,G_1\cup\{j\}}(b_j, b_{-j}))[2^{-1}\alpha(e_{j,G_2}(b_j) + e_{j,G_1\cup\{l\}}(b_j)) - \beta]p(b_{-j}|b_j) \geq 0.$$

Note that from the point of view of an arbitrary type $b_i \in G$, $A_{i,G}(b_i, b_{-i})$ accounts for others' types average performance in G.[36]

Intuitively, types in G_1 should be, "on average", sufficiently productive for them not to have incentives to move to G_2, and the contrary must happen to types in G_2. Finally, we emphasize that in general a stable AD-partition may involve situations in which all the types of an individual select the same group (pooling equilibrium) but also situations in which different types select different groups ((semi)separating equilibrium). For instance, if it is the case that for each pair of individuals $i < j$ each of the types of individual i is more productive than each of the types of individual j, then a stable AD-partitions may be the result of a situation in which all the types of an individual select the same group.

[36] For types in G_1 others' average performance is higher in G_1 and if they move to G_2 and the contrary happens for types in G_2.

Finally, note that if the productivity of types in G_1 is (weakly) higher than $\underline{b}(\alpha, \beta)$ then the AD-profile is stable according to Definition 6.

2. Analogously, we say that a profile of group choices is a RD-profile when for each type profile we have the configuration of groups described in Example 3. The analysis of stability in this case is also qualitatively analogous to the one in the main body, in particular, we must impose that for each type profile b the productivity of types that are not in $\mathcal{I}'_1$ is smaller than $\underline{b}(\alpha, \beta)$.

3. Note that apart from these two extreme scenarios in which for each type profile there is the same pattern of absolute or relative dominance between groups, we may have situations in which in a profile of group choices we do not always observe the same pattern. The stability analysis in this case is left for further research.

4. Finally, the social welfare analysis is parallel to the one in the main body. In particular, regardless of whether we define social welfare as the sum of individual utilities or as the sum of exerted efforts, it is maximized when for each type profile groups are consecutive.

7.3 More than two groups

We comment here on the case in which there are more than two groups to which individuals may belong. Notice that for an arbitrary individual i conditions (i) and (ii) in point 1 of Proposition 1 are incompatible, in words, if an individual is not willing to move to a group in which others' average performance is smaller than in her current group then she will in fact have incentives to move to a group in which others' average performance is higher than in her current group. The implication of this result is that the maximum number of groups all of which can be ranked according to absolute dominance is two. More generally, a partition is stable only if groups are such that each individual faces either higher or lower average performance in all the alternative groups she would potentially move to, but not both at the same time. This implies that when there are more than two groups any stable partition must involve a mixing of individual productivities.

Under Definition 2 of stability allowing for free mobility of individuals the requirement above that each individual faces either higher or lower average performance in all the alternative groups actually strengthens the requirements for a partition to be stable. However, under Definition 6 some of the individuals who would be willing to move to a group in which they face a higher average performance will be vetoed by the current members of such a group. In this case, partitions exhibiting a mixing of productivities may arise as stable, as the following example shows.

Example 6. Consider the setup of Example 4 and add two individuals that are now the two most productive, we have $(b_1, b_2, b_3, b_4, b_5, b_6, b_7) = (b_1, b_2, 0.6, 0.5, 0.27, 0.26, 0.25)$. The new groups are $G'_1 = \{3, 4\} \cup \{6\}$, $G'_2 = \{5\} \cup \{7\}$, $G'_3 = \{1, 2\}$. Let $b_1, b_2 > 0.6$ and then notice that none of the individuals in G'_3 are willing to move to the alternative groups since they are productive enough. Moreover, we already showed in Example 4 that individuals in G'_1 and G'_2 do not have incentives to move between these two groups. Finally, note that under Definition 6 of stability, individual movements from G'_1 or G'_2 to G'_3 are vetoed by individuals 1 and 2 since they both suffer utility losses when a (less productive) individual intends to move to such a group. Thus, this partition is stable.

Note again that individuals in G'_1 and G'_2 do not have incentives to move between these two groups, and thus, as we already illustrated in Example 4, some of these individuals prefer to be big fishes in a relative small pond and some others prefer to be small fishes in a relatively high pond.

7.4 Existence of stable partitions (in pure strategies)

We consider here Definition 6 of stability to discuss about the existence of stable partitions:

1. A stable AD-partition $\mathcal{G} = \{G_1, G_2\}$ exists for any parameter configuration $(b_i)_{i \in \mathcal{N}}$, α and β such that $b_i > \underline{b}(\alpha, \beta)$ for individuals in G_1. In this case the number of individuals that would allow the adhesion of a new member to G_1 is not sufficient since in fact all of them suffer utility losses when others' average performance decreases and that is the case when a less productive member moves to G_1.

2. Consider an RD-partition $\mathcal{G} = \{G_1, G_2\}$, as the one described in Example 3. Notice that $|\mathcal{I}_1| = \lceil |G_1|/2 \rceil$. Let $(b_i)_{i \in \mathcal{N}}$, α and β be such that $b_i > \underline{b}(\alpha, \beta)$ for each $i \in \mathcal{I}_1$. In this case, the movement to G_1 of any $i \in \mathcal{J}_2$ is vetoed by all the individuals in $\mathcal{I}_1$. The reason is the one presented above: this adhesion lowers the average performance that each of the individuals in $\mathcal{I}_1$ faces. Notice also that none of these individuals has incentives to move to G_2. It then follows that for such a RD-partition to be stable (i) either condition 2 of Lemma 3 is satisfied for individual $j \in \mathcal{J}_1$, or it is not satisfied but individuals in $\mathcal{I}_1$ veto her adhesion and (i) condition 1 of Lemma 3 needs to be checked only for the individuals in $\mathcal{I}_2$ and also

Regarding (i), it is intuitive that when individual $j \in \mathcal{J}_1$ is of sufficiently low productivity, the average performance from the point of view of individuals in $\mathcal{I}_1$ would decrease in $G_1 \cup \{j\}$, as an illustration, using the setup of Example 4, for individuals 1 and 2, who belong to G_1, $A_{1,G_1} = 0.62 > A_{1,G_1 \cup \{3\}} = 0.47$ and $A_{2,G_1} = 0.74 > A_{2,G'_1 \cup \{3\}} = 0.55$.

In line with the discussion above the following result establishes how equilibrium efforts are shaped when an individual moves to a group. With this information, we can keep track of how average performance is affected and then offer sharper insights regarding the stability of a RD-partition, as it will be made clearer below. For this purpose let e^{+j} the vector of equilibrium efforts of individuals in $G_1 \cup \{j\}$ and e the augmented vector of equilibrium efforts of individuals in G_1 where an entry equal to zero is introduced in the position that individual j would occupy, according to the ranking of productivities.

Lemma 7. Consider a RD-partition as the one described in Example 3 and let $\Delta_j e = e^{+j} - e$. It then follows that

$$\frac{(\partial \Delta_j e)_i}{\partial b_j} > 0$$

for each $i \in G_1 \cup \{j\}$.

The equilibrium efforts of individuals in $G_1 \cup \{j\}$ are increasing in b_j because on the one hand the efforts contained in e, which are computed before individual j moves to the group, are not affected by the productivity of individual j, and on the other hand, the efforts contained in e^{+j} all increase when the productivity of j increases. Then, the smaller b_j the easier is that the average performance from the point of view of individuals in $\mathcal{I}_1$ decreases when j moves to the group. Hence, such individuals will veto the adhesion of individual j.

In what follows we illustrate this intuition for a RD-partition. In Table 1, the RD-partition described in Example 3 is stable for the parameters considered, when $b_3 = \{0.21, 0.4, 0.5\}$. Individual 4 does not have incentives to move to G_2 as $e_{4,G_1} = 2.5 > 2\beta/\alpha = 2$, thus condition 1 of Lemma 4 is satisfied for her. Further, individuals 1 and 2 are sufficiently productive and thus they do not have incentives to move to G_2 and neither allow the adhesion of individuals 3 to their group G_1, since this lowers all the efforts and hence the average performance 1 and 2 face. They thus neither allow the adhesion of individual 5.

$\Delta_3 e$	$b_3 = 0.21$	$b_3 = 0.4$	$b_3 = 0.5$	$b_3 = 0.6$	$b_3 = 0.69$
$e_{1,G_1\cup\{3\}} - e_{1,G_1}$	-0.46	-0.2	-0.02	0.19	0.4
$e_{2,G_1\cup\{3\}} - e_{2,G_1}$	-0.52	-0.25	-0.06	0.16	0.37
$e_{3,G_1\cup\{3\}} - e_{3,G_1}$	1.68	2.21	2.52	2.85	3.13
$e_{4,G_1\cup\{3\}} - e_{4,G_1}$	-0.82	-0.51	-0.3	-0.03	0.2
e_{4,G_1}	2.5	2.5	2.5	2.5	2.5

Table 1 – Differences in equilibrium efforts for increasing values of b_3, $(b_1, b_2, b_3, b_4, b_5) = (0.8, 0.7, b_3, 0.2, 0.1)$ and $\alpha = \beta = 1$

Regarding (ii), we provide a sufficient condition for individuals in $\mathcal{I}_2$ not to have incentives to abandon their own group.

Lemma 8. Consider a RD-partition described in Example 3 and let $b_j \geq \underline{b}(\alpha, \beta)$ for each $j \in \mathcal{I}_1$. Then, if for the least productive individual $k \in \mathcal{I}_2$ it holds that

$$b_k(1+\beta) \geq \frac{\beta}{\alpha}\left[2 - \frac{\alpha \sum_{i \in \mathcal{I}_1} b_j}{|G_1| - 1}\right], \tag{6}$$

then no individual in $\mathcal{I}_2 \in G_1$ has incentives to move to G_2.

If group quality is sufficiently relevant with respect to local standing (low β/α) or individuals in $\mathcal{I}_1$ are sufficiently productive, individuals in $\mathcal{I}_2$ would have fewer incentives to abandon their own group. In both cases, the right-hand side of Eq. (6) tends to be small.

The results in Lemma 7 and Lemma 8 allow us to conclude that (i) the higher the productivities of the individuals in $\mathcal{I}_1$, who are also the most productive individuals in the society and (ii) the closer $j \in \mathcal{J}_1$ is to the most productive individual in $\mathcal{I}_2$, the easier is that the conditions that ensure the stability of the proposed RD-partition are met.

7.5 Efficient efforts: existence and a tax/subsidy scheme to restore them

The following Lemma 9 states when efficient efforts exist, as we did for Nash equilibrium efforts. For a group G of cardinality $|G|$, let V be a square matrix of size $|G|$ with entries: $v_{ii} = 0$ and $v_{ij} = (b_i + b_j)/(|G| - 1)$, $j \neq i$ and denote by $\delta_1(V)$ its largest eigenvalue. Let I an identity matrix of size $|G|$, and b be the vector of private productivities.

Lemma 9. The matrix $[I - \alpha V]^{-1}$ is well-defined and non-negative if and only if $1 > \alpha\delta_1(V)$. Then, efficient efforts are uniquely characterized by

$$e_G^E = (I - \alpha V)^{-1} b.$$

For each pair of individuals $i, j \in G$ such that $b_i > b_j$ it holds that $e_{i,G}^E > e_{j,G}^E$ and, as a consequence, $b_i e_{i,G}^E > b_j e_{j,G}^E$.

We also comment on the possibility of introducing per-unit taxes/subsidies to restore efficient efforts. In particular, suppose that we add a stage before the effort game is played, in which a planner announces a per unit of effort tax/subsidy. In this case individuals choose effort by internalizing such a tax/subsidy scheme, and that would induce the choice of efficient efforts as a result. The scheme consists on giving each agent $i \in G$ the following tax/subsidy per unit of effort

$$S_i^E = b_i \left[\frac{\alpha \sum_{j \neq i} e_{j,G}^E}{|G| - 1} - \beta\right]. \tag{7}$$

The expression in Eq. (7) results when individual i chooses effort to maximize the utility function

$$u_i(e_{i,G}, e_{-i,G}) = [b_i + S_i^E]e_{i,G} - \frac{1}{2}e_{i,G}^2 + \alpha[e_{i,G}\, A_{i,G}] - \beta[A_{i,G} - b_i e_{i,G}],$$

The tax/subsidy scheme suggests that it is necessary to subsidize individuals when others in their own group exert an average effort above β/α, as in this case efficient efforts are above Nash equilibrium efforts, and tax them in the opposite case, as it follows that Nash equilibrium efforts are above efficient efforts. Thus, there are particular situations in which to increase welfare, efforts should even be taxed. That may happen in particular when local standing concerns are very important and group quality concerns, on the contrary, are not.[37]

Example 7. As a follow-up to Example 5 recall that Nash equilibrium efforts are $e_{1,G} = 1.9$ and $e_{2,G} = 1.56$ whereas efficient efforts are smaller for each individual and equal to $e_{1,G}^E = 1,62$ and $e_{2,G}^E = 1.37$, respectively. Thus both individuals are over-exerting effort we they make decisions unilaterally and these actions impose negative externalities on others (α is sufficiently small and β is sufficiently high). As a consequence, both individuals should be taxed to restore efficient efforts. In particular, we would have that $S_1^E = -0.25 < 0$ and $S_2^E = -0.08 < 0$. Upon an increase group quality concerns through a higher $\alpha' = 0.57$, efficient efforts are above Nash equilibrium efforts, in particular, $e_{1,G}^E = 2.01 > e_{1,G} = 1.97$ and $e_{2,G}^E = 1.77 > e_{2,G} = 1.68$. In this case individuals should be subsidized, that is, $S_1^E = 0.007 > 0$ and $S_2^E = 0.06 > 0$.

7.6 Proofs

Proof of Lemma 1. Consider a group G of size $|G| > 1$. The best reply $e_{i,G}(e_{-i,G}) = b_i(1+\beta) + \alpha A_{i,G}$ of each $i \in G$ can be expressed in matrix form as

$$e_G = Bb + \alpha W e_G, \tag{8}$$

where $\alpha > 0$ is a scalar, e_G is the $|G| \times 1$ vector of efforts, b is the $|G| \times 1$ vector of productivities, and B is a square diagonal matrix of size $|G|$ such that entry $b_{jj} = 1 + \beta$. Finally, W is the square matrix of size $|G|$ with entries: $w_{ii} = 0$ and $w_{ij} = b_j/(|G| - 1)$, for each $j \neq i$. By Theorem 6.2.24 in Horn and Johnson (2013) and Theorem III* in Debreu

[37] See Helsley and Zenou (2014) and Ushchev and Zenou (2020) for an analysis of taxes/subsidies on effort choices and Langtry (2023) for related intuitions in the context of consumption choices and local comparisons.

and Herstein (1953), the system of equations above has a unique solution described by

$$e_G = (I - \alpha W)^{-1} Bb$$

if and only if $1 > \alpha\mu_1(W)$, where $\mu_1(W)$ is the largest eigenvalue of W. In such a solution, for each pair $i, j \in G$ such that $b_i > b_j \Rightarrow b_i e_{i,G} > b_j e_{j,G}$. Consider

$$e_{j,G}(e_{-j,G}) = b_j(1+\beta) + (|G|-1)^{-1}\alpha\big[b_i e_{i,G} + \sum_{k \neq i,j} b_k e_{k.G}\big],$$

and

$$e_{i,G}(e_{-i,G}) = b_i(1+\beta) + (|G|-1)^{-1}\alpha\big[b_j e_{j,G} + \sum_{k \neq i,j} b_k e_{k.G}\big].$$

It follows that $e_{j,G}(e_{-j,G}) - e_{i,G}(e_{-i,G}) = (b_j - b_i)(1+\beta) + \alpha(|G|-1)^{-1}[b_i e_{i,G_s} - b_j e_{j,G}]$.

Assume, contrary to our statement, that $b_j e_{j,G} \geq b_i e_{i,G}$. In this case the right-hand side of the previous equality is negative. For the left-hand side to be negative we require that $e_{i,G} > e_{j,G}$ but this implies the contradiction that $b_j e_{j,G} < b_i e_{i,G}$. Thus, it must be the case that $b_i e_{i,G} > b_j e_{j,G}$. ∎

Proof of Proposition 1. We prove points 1 to 3 of Proposition 1. Consider a group $G_s \in \mathcal{G}$, $s \in \{1, 2\}$.

Points 1 and 2. The utility of individual i in $G_s \in \mathcal{G}$ when she plays her best reply $e_{i,G_s} = b_i(1+\beta) + \alpha A_{i,G_s}$ can be rewritten, using Eq. (2), as

$$2^{-1} e^2_{i,G_s} - \beta A_{i,G_s}. \tag{9}$$

Her utility if she moves to group $G_{s'} \in \mathcal{G}$, $s' \in \{1, 2\}$, $s' \neq s$ is

$$2^{-1} e^2_{i,G_{s'}\cup\{i\}} - \beta A_{i,G_{s'}\cup\{i\}}. \tag{10}$$

Let $A_{i,G_{s'}\cup\{i\}} \leq A_{i,G_s}$. It then follows that $e_{i,G_{s'}\cup\{i\}} < e_{i,G_s}$ and thus i does not have incentives to move to $G_{s'}$ if and only if Eq. (9) $\geq$ Eq. (10), that is, $e^2_{i,G_s} - e^2_{i,G_{s'}\cup\{i\}} \geq 2\beta[A_{i,G_s} - A_{-i,G_{s'}\cup\{i\}}]$. This expression is equivalently rewritten as

$$e_{i,G_s} + e_{i,G_{s'}\cup\{i\}} \geq \frac{2\beta}{\alpha}. \tag{11}$$

By analogous reasoning, an individual j in G_s is not willing to move to a group $G_{s'}$ such that $A_{j,G_s} \leq A_{j,G_{s'}\cup\{j\}}$ if and only if

$$e_{j,G_s} + e_{j,G_{s'}\cup\{j\}} \leq \frac{2\beta}{\alpha}. \tag{12}$$

Notice that Eq. (12) is violated for any j such that $b_j > \beta/\alpha(1+\beta)$ since in this case, we have that $e_{j,G_s} > \beta/\alpha$, and $e_{j,G_{s'}\cup\{j\}} > \beta/\alpha$. Thus, such an individual can only face a smaller average performance in a group different from her own.

Point 3. Use the best reply of an individual $i \in G$, $|G| > 1$ to write the equilibrium utility as $2^{-1}[b_i(1+\beta) + \alpha A_{i,G}]^2 - \beta A_{i,G}$. Recall that the utility of i when she is the only member of a group is $2^{-1}b_i^2$. Consider then the function $f(x) = 2^{-1}[b_i(1+\beta) + \alpha x]^2 - \beta x - 2^{-1}b_i^2$ or equivalently $f(x) = 2^{-1}\alpha^2 x^2 + (\alpha b_i(1+\beta) - \beta)x + 2^{-1}b_i^2[(1+\beta)^2 - 1]$. This function takes a positive value at $x = 0$ and is non-decreasing in x if and only if $f'(x) = \alpha^2 x^2 + \alpha b_i(1+\beta) - \beta > 0$, thus, $b_i \geq \beta/\alpha(1+\beta)$ suffices for $f(x) > 0$ whenever $x \geq 0$. ∎

Proof of Lemma 2. Consider a partition $\mathcal{G} = \{G_1, G_2\}$ composed of non-singleton groups, where G_1 absolutely dominates G_2. In this case, any individual in G_1 faces a smaller average performance if she moves to G_2, and the contrary happens to any individual in G_2 who moves to G_1. We prove the formal statement below.

According to the proof of Lemma 1 we have that $e_G = (I - \alpha W)^{-1}Bb$. For $\alpha\mu_1(W) < 1$, we argued that $(I-\alpha W)^{-1}$ is well-defined. Note that $(I-\alpha W)^{-1}$ can be equivalently written, using the Neumann series expansion, as $T \equiv \sum_{k=0}^{\infty} \alpha^k W^k$. Let t_{ij} be an arbitrary ij entry of T. We have that $t_{ij} = \sum_{k=0}^{\infty} \alpha^k w_{ij}^{[k]}$, where $w_{ij}^{[k]}$ is ij the entry of W^k.

Consider two groups, $\overline{G}$ of cardinality n and $\underline{G}$ of cardinality m, where n and m are not necessarily equal. Set, for simplicity, $|\overline{G}|, |\underline{G}| > 2$.[38]

We first prove that when each non-zero entry in a $n \times n$ productivity matrix $\overline{W}$ associated to $\overline{G}$ is strictly higher than any other non-zero entry in a $m \times m$ productivity matrix $\underline{W}$ associated to $\underline{G}$, then each entry $\overline{w}_{ij}^{[k]}$ of $\overline{W}^k$ is (weakly) higher than any other entry $\underline{w}_{hl}^{[k]}$ in $\underline{W}^k$. We then use this result to conclude that equilibrium efforts are the highest in $\overline{G}$. In doing so, it is important to recall the expression above for Neumann series expansion of $(I - \alpha W)^{-1}$, upon which equilibrium efforts are characterized. Consider that a productivity matrix $\overline{W} = (n-1)^{-1}\overline{S}$, where $\overline{S}$ has each non-zero generic entry $\overline{s}_{ij}$ (weakly) higher than any other non-zero entry $\underline{s}_{ij}$ of a matrix $\underline{S}$ such that $\underline{W} = (m-1)^{-1}\underline{S}$. We have that $\overline{w}_{ij}^{[2]} = 1/(n-1)^2 \sum_{k=1}^{n} \overline{s}_{ik}\overline{s}_{kj}$. Analogously, for $\underline{\mathrm{W}}$ consider $\underline{w}_{hl}^{[2]} = 1/(m-1)^2 \sum_{k=1}^{m} \underline{s}_{hk}\underline{s}_{kl}$. Notice then that each entry in $\sum_{k=1}^{n} \overline{s}_{ik}\overline{s}_{kj}$ is (weakly) higher than each entry in $\sum_{k=1}^{m} \underline{s}_{hk}\underline{s}_{kl}$. That implies that $\overline{w}_{ij}^{[2]} > \underline{w}_{hl}^{[2]}$ $\forall i,j,h,l$. Consider now $\overline{W}^k$ for $k > 1$. We have that $\overline{w}_{ij}^{[k]} = 1/(n-1)^k \sum_{k=1}^{n} \overline{s}_{ik}^{[k-1]}\overline{s}_{kj}$.[39] Analogously, for $\underline{W}^k$ we have $\underline{w}_{hl}^{[k]} = 1/(m-1)^k \sum_{k=1}^{n} \underline{s}_{hk}^{[k-1]}\underline{s}_{kl}$.

[38] Below we comment on the trivial case in which some group has cardinality two.

[39] Notice that there are $(n-1)^k$ elements in such a sum, $(n-1)^{k-1}$ for column i and row i and $(n-2)(n-$

Let each element in $\sum_{k=1}^{n} \overline{s}_{ik}^{[k-1]} \overline{s}_{kj}$ be higher than each of the elements in $\sum_{k=1}^{n} \underline{s}_{hk}^{[k-1]} \underline{s}_{kl}$. Thus, by the same reasoning as above, each entry ij in $\overline{W}^{k+1}$ is higher than any other entry hl in $\underline{W}^{k+1}$.

Let $\overline{G}$ be a group defined by the above productivity matrix $\overline{W}$. By previous arguments, each entry in $\overline{W}^k$ is higher than any other entry $\underline{W}^k$ for each k. As the vector $\overline{b}$ of productivities associated to $\overline{G}$ has each entry weakly higher than the vector $\underline{b}$ of productivities associated to $\underline{G}$, we then conclude that for any $i \in \overline{G}$ the equilibrium effort $e_{i,\overline{G}} = \sum_j ((I - \alpha \overline{W})^{-1} \beta \overline{b})_{ij}$ (see Eq. 7.6) is higher than the equilibrium effort $e_{j,\overline{G}} = \sum_j ((I - \alpha \underline{W})^{-1} \beta \underline{b})_{ij}$ of any $j \in \underline{G} \cup \{i\}$. Thus the private product of each individual in $\overline{G}$ is higher than the private product of any other individual in $\underline{G} \cup \{i\}$. Therefore from the point of view of $i \in \overline{G}$ average performance is the highest in her current group. The case in which an individual $i \in \underline{G}$ is considering moving to $\overline{G}$ operates in an analogous way (in the natural opposite direction).[40]

Using the result in point 1 of Proposition 1 together with the insights above it is direct to assess that the partition $\mathcal{G}$ defined above is stable if and only if Eq. (11) holds for each $i \in G_1$ and Eq. (12) holds for each $j \in G_2$. ∎

Proof of Proposition 2. Using the utility specification in Eq. (2) we have that

$$\sum_{i \in G} u_i(e_{i,G}, e_{-i,G}) = \sum_{i \in G} \left[b_i \, e_{i,G} - \frac{1}{2} \, e_{i,G}^2 + \alpha [e_{i,G_s} \, A_{i,G}] - \beta [A_{i,G} - b_i \, e_{i,G_s}] \right].$$

Note that $\sum_{i \in G} [A_{i,G} - b_i e_{i,G}] = 0$ and thus

$$\sum_{i \in G} u_i(e_{i,G}, e_{-i,G}) = \sum_{i \in G} e_{i,G} \left[b_i - \frac{1}{2} e_{i,G} + \alpha A_{i,G} \right]. \tag{13}$$

The efforts that maximize the sum of individual utilities are such that for each individual i

$$e_{i,G}^E = b_i \left[1 + \frac{\alpha \sum_{j \neq i} e_{j,G}^E}{|G| - 1} \right] + \alpha A_{i,G}^E. \tag{14}$$

Plugging Eq. (14) into Eq. (13) we get

$1)^{k-1}$ for row i and column $j \neq i$.

[40] When a group, say $\overline{G}$, has cardinality two, we need to take into account that some of the entries of the productivity matrix (and its subsequent powers of order k) are zero -these are the off-diagonal entries when k is odd and the diagonal entries when k is even-. In this case, the proof follows as well. The result simply states that in the power of the productivity matrix associated to a group $\overline{G}$, each non-zero entry is higher than any other non-zero entry in the corresponding power of the productivity matrix associated to $\underline{G}$.

$$2^{-1}\sum_{i\in G} e^E_{i,G}\left[b_i + \alpha\frac{\sum_{j\neq i}(b_j-b_i)e^E_{j,G}}{|G|-1}\right].$$

In the expression above, $\sum_{i\in G} e^E_{i,G}\sum_{j\neq i}(b_j-b_i)e^E_{j,G}=0$ because for each pair $i,j\in G$ the expression $-b_i e^E_{i,G}e^E_{j,G}$, which is negative from the point of view of i, enters with a positive sign for individual j. Thus the sum of individual utilities in G amounts to $2^{-1}\sum_{i\in G} b_i e^E_{i,G}$. ∎

Proof of Lemma 3. By analogous reasoning than the one in the the proof of Lemma 2 it also follows that if for two groups of equal cardinality, each entry ij in the productivity matrix associated to one group is weakly higher than its counterpart entry in the alternative group, then the equilibrium efforts in the former group are higher than in the latter group.

Consider the RD-partition of Example 3. For each individual $i\in G_1$ it follows that $|G_1|=|G_2+1|$. Notice that $|G_2+1|$ is the cardinality of $G_2\cup\{i\}$ and that each individual in G_1 has (weakly) higher productivity than the individual that occupies the same position in terms of productivity in $G_2\cup\{i\}$. Thus, the individuals who belong to G_1 enjoy a higher average performance in their group than if they move to G_2. In this case, condition in point 1 of Lemma 3 must hold for each of these individuals.

For $j\in\mathcal{J}_1\in G_2$ it follows that $|G_1+1|>|G_2|$. Consider the $|G_2|$ least productive individuals in $G_1\cup\{j\}$ and note that for each $j\in\mathcal{J}_1\in G_2$ and relative to each individual in G_2, the individual that occupies the same position among the least $|G_2|$ productive individuals in $G_1\cup\{j\}$, is (weakly) more productive. Thus, we would be able to already conclude, using analogous arguments than above, that $j\in\mathcal{J}_1\in G_2$ faces a higher average performance in $G_1\cup\{j\}$ than in her current group G_2. Moreover, there are still $|G_1+1|-|G_2|$ who are more productive than the $|G_2|$ least productive individuals in $G_1\cup\{j\}$ and it also follows, by analogous reasoning than the one in the proof of Lemma 2, that when we add to a group G individuals that are more productive than the ones already in such a group, then efforts (and therefore private products) of the individuals originally in G increase. Thus definitely $j\in\mathcal{J}_1\in G_2$ faces a higher average performance in $G_1\cup\{j\}$ than in G_2. In this case condition in point 2 of Lemma 3 must hold for her.

For individuals in $\mathcal{J}_2$ we cannot make the same type of claims above regarding the pairwise comparisons of productivities. Thus, either of the conditions 1 or 2 in Lemma 3 must hold for each individual in $\mathcal{J}_2$ depending on whether she is facing respectively, a lower or a higher average performance in the group she evaluates whether to move. ∎

Proof of Lemma 4. Efficient efforts in Eq. (4) of Proposition 2 are expressed in matrix form as

$$e_G^E = b + \alpha V e_G^E, \tag{15}$$

where $\alpha > 0$ is a scalar, e_G^E is the $|G| \times 1$ vector of efforts, b is the $|G| \times 1$ vector of productivities and V is the square matrix of size $|G|$ with entries: $v_{ii} = 0$ and $v_{ij} = (b_i + b_j)/(|G| - 1)$, for each $j \neq i$.

Let O be a square matrix of size $|G|$ such that an arbitrary row i consists of the entries: $o_{ii} = 0$ and $o_{ij} = b_i/(|G| - 1)$, for each $j \neq i$. Recall that $O = W'$. Notice that $V = W + W'$ where W is the square matrix defined in Lemma 1.

It is direct to observe that the expression in Eq. (8) and the expression in Eq. (15) coincide, and therefore the vector e of efficient and Nash equilibrium efforts coincide, whenever $b + \alpha O e = Bb$. Using such an equality, for each $i \in G$ we have

$$\frac{\alpha \sum_{j \neq i} e_j}{|G| - 1} = \beta.$$

Such an expression simultaneously holds for each $i \in G$ whenever all individuals exert an effort level of β/α. Thus, suppose that there is an effort profile that is simultaneously efficient and a Nash equilibrium. Then, using the efficient efforts in Eq. (4) or the effort's best reply, it must hold that for each $i \in G$

$$\frac{\beta}{\alpha} = b_i[1 + \beta] + \beta \frac{\sum_{j \neq i} b_j}{|G| - 1}, \tag{16}$$

or equivalently

$$\frac{\beta}{1 + \beta} = \frac{\alpha b_i}{1 - \alpha \frac{\sum_{j \neq i} b_j}{|G| - 1}}. \tag{17}$$

As the left-hand side is a constant, it follows that for each pair $i, j \in G$ we have

$$\frac{\alpha b_i}{1 - \alpha \frac{\sum_{m \neq i} b_m}{|G| - 1}} = \frac{\alpha b_j}{1 - \alpha \frac{\sum_{k \neq j} b_k}{|G| - 1}}.$$

The expression above implies that

$$b_i[|G| - 1 - \alpha \sum_{k \neq j} b_k] = b_j[|G| - 1 - \alpha \sum_{m \neq i} b_m] = (b_i - b_j)[|G| - 1] = \alpha[b_i \sum_{k \neq j} b_k - b_j \sum_{m \neq i} b_m],$$

or equivalently

$$(b_i - b_j)[|G| - 1] = \alpha[b_i^2 - b_j^2] + \alpha(b_i - b_j) \sum_{s \neq i, j} b_s.$$

We then have that for an arbitrary individual i

$$|G| - 1 - \alpha \sum_{j \neq i} b_j = \alpha b_i. \tag{18}$$

According to Eq. (18) the right-hand side of Eq. (17) equals $|G| - 1 \geq 1$ and that is contradiction since its left-hand side is smaller than one. Thus, we conclude that no strategy profile in which all efforts are equal to β/α can be efficient and a Nash equilibrium.

It has already been stated above that both efficient and Nash efforts are the same if and only if $b + Oe = Bb$ and we also argued that this equality led to an impossibility. Thus, for at least one individual, the efficient effort and the Nash efforts cannot be the same. The difference $Oe_G^E - (B-I)b$ measures the discrepancy between two such vectors of effort. Using $e_G^E = (I - \alpha V)^{-1}b$ we have that the difference between the two vectors of efforts, $e_G^E - e_G$, must amount to $O(I-\alpha V)^{-1}b-(B-I)b$. Using the Newman series decomposition we rewrite this expression as

$$[O \sum_{k=0}^{\infty} \alpha^k V^k - (B - I)]b,$$

and thus conclude that the difference $e_G^E - e_G$ increases as: α increases, β decreases and individuals are more productive. ∎

Proof of Lemma 5. Consider that an AD-partition $\mathcal{G} = \{G_1, G_2\}$, where $|G_1|, |G_2| > 1$, is efficient, and recall that individuals in such a partition exert efficient efforts described in Eq. (4). In this case the utility that accrues to $i \in G_s$, $s \in \{1, 2\}$ amounts to $2^{-1}(e_{i,G_s}^E)^2 - \beta A_{i,G_s}^E + b_i e_{i,G_s}^E(\beta - \alpha \bar{e}_{i,G_s}^E)$. An individual $i \in G_1$ does not have incentives to move to G_2 if and only if

$$2^{-1}(e_{i,G_1}^E)^2 - \beta A_{i,G_1}^E + b_i e_{i,G_1}^E(\beta - \alpha \bar{e}_{i,G_1}^E) \geq 2^{-1}(e_{i,G_2 \cup \{i\}}^E)^2 - \beta A_{i,G_2\cup\{i\}}^E + b_i e_{i,G_2\cup\{i\}}^E(\beta - \alpha \bar{e}_{i,G_2\cup\{i\}}^E). \tag{19}$$

Note that individuals in G_2 are each of smaller productivity than individuals in G_1, thus recalling that efficient efforts within a group are computed according to the expression in Lemma 9 where matrix V has higher entries when we consider group G_1 than when we consider G_2, we use analogous arguments than the ones in the proof of Lemma 2 to conclude than the efforts exerted by the members in G_1 are higher than the ones exerted by the members of $G_2 \cup \{i\}$. Also, by the proof of Lemma 9, efficient efforts preserve the order of individual productivities. These observations allow us to conclude that $\bar{e}_{i,G_1}^E > \bar{e}_{i,G_2\cup\{i\}}^E$ and also that $A_{i,G_1}^E > A_{i,G_2\cup\{i\}}^E$. Thus, using the expression for efficient efforts in Eq. (4) and after some direct algebra we conclude that Eq. (19) is equivalent to requiring that

$$e^E_{i,G_1} + e^E_{i,G_2\cup\{i\}} + 2b_i\left[\beta\frac{(\bar{e}^E_{i,G_1} - \bar{e}^E_{i,G_2\cup\{i\}})}{e^E_{i,G_1} - e^E_{i,G_2\cup\{i\}}} + \frac{e^E_{i,G_1}(\beta - \alpha\bar{e}^E_{i,G_1}) - e^E_{i,G_2\cup\{i\}}(\beta - \alpha\bar{e}^E_{i,G_2\cup\{i\}})}{e^E_{i,G_1} - e^E_{i,G_2\cup\{i\}}}\right] > 2\frac{\beta}{\alpha},$$

.

Let $\bar{e}^E_{i,G_1\&G_2\cup\{i\}} = (\bar{e}^E_{i,G_1}, \bar{e}^E_{G_2\cup\{i\}})$ and analogously for $e^E_{i,G_1\&G_2\cup\{i\}}$. Then, we equivalently write

$$e^E_{i,G_1} + e^E_{i,G_2\cup\{i\}} + 2b_i\theta(\bar{e}^E_{i,G_1\&G_2\cup\{i\}}, e^E_{i,G_1\&G_2\cup\{i\}}) > 2\beta\left[\frac{1}{\alpha} - \frac{b_i(\bar{e}^E_{i,G_1} - \bar{e}^E_{i,G_2\cup\{i\}})}{e^E_{i,G_1} - e^E_{i,G_2\cup\{i\}}}\right],$$

where $\theta(\bar{e}^E_{i,G_1\&G_2\cup\{i\}}, e^E_{i,G_1\&G_2\cup\{i\}})$ is expressed as in Lemma 5. The case of individuals in G_2 is analogous and hence omitted.

∎

Proof of Proposition 4. Consider the two groups in which individuals may be organized. The sum of Nash equilibrium efforts is

$$\sum_{G\in\mathcal{G}}\sum_{i\in G} e_{i,G} = (1+\beta)\sum_{i\in G\,:\,|G|>1} b_i + \sum_{i\in G\,:\,|G|=1} b_i + \alpha\sum_{G\,:\,|G|>1}\sum_{i\in G} A_{i,G},$$

where

$$\sum_{G\,:\,|G|>1}\sum_{i\in G} A_{i,G} = \sum_{G\,:\,|G|>1}\sum_{i\in G} b_i e_{i,G}.$$

The Nash equilibrium effort that each individual exerts in a non-singleton group is increasing in that individual's productivity and also, the more productive the individual is the more she is sensitive to other's productivities. The proof of this last part is qualitatively analogous to the one presented in the proof of Proposition 2, and hence omitted. Thus, we conclude that partitions that maximize the sum of individual efforts consist of consecutive groups and that a singleton group must be composed of the least productive individual. ∎

Proof of Lemma 6. In a stable partition $\mathcal{G}$, for $i \in G_s \in \mathcal{G}$ it holds that $u_i(e_{i,G_s}, e_{-i,G_s})m(T_{G_{s'}}) - u_i(e_{i,G_{s'}\cup\{i\}}, e_{-i,G_{s'}\cup\{i\}})m(T_{G_s\setminus\{i\}}) \geq 0$, $s, s' \in \{1,2\}$, $s \neq s'$ and $m \in \{f, g\}$.

Let $u_i(e_{i,G_s}, e_{-i,G_s}) > 0$ and $u_i(e_{i,G_{s'}\cup\{i\}}, e_{-i,G_{s'}\cup\{i\}}) > 0$. Recall that in the model with no externalities, the condition is equivalent to require that

$$\frac{u_i(e_{i,G_s}, e_{-i,G_s})}{u_i(e_{i,G_{s'}\cup\{i\}}, e_{-i,G_{s'}\cup\{i\}})} \geq 1, \tag{20}$$

whereas in the model with externalities, we equivalently have

$$\frac{u_i(e_{i,G_s}, e_{-i,G_s})}{u_i(e_{i,G_{s'}\cup\{i\}}, e_{-i,G_{s'}\cup\{i\}})} \geq \frac{m(T_{G_s\setminus\{i\}})}{m(T_{G_{s'}})}. \tag{21}$$

It is then useful to consider two cases

1. If $T_{G_{s'}} < T_{G_s\setminus\{i\}}$ then under positive externalities $m = f$ and $f(T_{G_{s'}}) < f(T_{G_s\setminus\{i\}})$, thus Eq. (21) is harder to be met than Eq. (20) and under negative externalities $m = g$ and $g(T_{G_{s'}}) > g(T_{G_s\setminus\{i\}})$, thus the contrary happens.

2. if $T_{G_{s'}} > T_{G_s\setminus\{i\}}$ then under positive externalities $m = f$ and $f(T_{G_{s'}}) > f(T_{G_s\setminus\{i\}})$, thus Eq. (21) is easier to be met than Eq. (20) and under negative externalities $m = g$ and $g(T_{G_{s'}}) < g(T_{G_s\setminus\{i\}})$, thus the contrary happens.

∎

Proof of Lemma 7. We analyze how equilibrium efforts change when a new individual moves to group G. Let Δ_j the operator that maps a vector or matrix into the values of this vector or matrix before and after an individual j moves to G. Thus $\Delta_j e = e^{+j} - e$ captures changes in equilibrium efforts and $\Delta_j W = W^{+j} - W$ captures changes in the matrix of weights after individual j moves to G. Note that $\Delta_j b$ is a vector of cardinality $|G|$ in which the only non-zero entry corresponds to individual j, thus this expression captures the inclusion of j in G.

Recall that $e = \alpha W e + Bb$ and note that, as individual j is not present initially in G, the entry corresponding to this individual in (the augmented row vector of cardinality $|G| + 1$) e takes value zero and W is an augmented matrix of cardinality $|G| + 1$ such that row j is full of zeros. Therefore, the difference in efforts before and after individual j moves to G can be expressed as

$$\Delta_j e = \alpha[W^{+j} e^{+j} - W e] + B\Delta_j b.$$

We rewrite the expression above, adding and subtracting the term $W^{+j} e$, as

$$\Delta_j e = \alpha[W^{+j} e^{+j} - W^{+j} e + W^{+j} e - W e] + B\Delta_j b.$$

In turn, we rewrite such an expression as

$\Delta_j e = \alpha[W^{+j}\Delta_i e + \Delta_j W e] + B\Delta_j b, \Rightarrow \Delta_j e = \alpha[(W + \Delta_j W)\Delta_j e + \Delta_j W e] + B\Delta_j b, \Rightarrow$ $\Delta_j e = \alpha W \Delta_j e + \alpha \Delta_j W \Delta_j e + \alpha \Delta_j W e + B\Delta_j b.$

We finally use $\Delta_j W = W^{+j} - W$ to end up with the expression

$$\Delta_j e = \alpha W^{+j} \Delta_j e + \alpha \Delta_j W e + B \Delta_j b,$$

which is equivalent to

$$\Delta_j e = (I - \alpha W^{+j})^{-1} [\alpha \Delta_j W e + B \Delta_j b]. \tag{22}$$

The derivative of $\Delta_j e$ in Eq. (22) with respect to b_j is

$$\frac{\partial \Delta_j e}{\partial b_j} = (I - \alpha W^{+j})^{-1} \left[\frac{1}{b_j} B \Delta_j b \right] + \frac{\partial (I - \alpha W^{+j})^{-1}}{\partial b_j} [\alpha \Delta_j W e + B \Delta_j b],$$

or equivalently

$$\frac{\partial \Delta_j e}{\partial b_j} = (I - \alpha W^{+j})^{-1} \left[\frac{1}{b_j} B \Delta_j b - \frac{\partial (I - \alpha W^{+j})}{\partial b_j} (I - \alpha W^{+j})^{-1} [\alpha \Delta_j W e + B \Delta_i b] \right].$$

Note that the last term $\alpha \Delta_j W e + B \Delta_i b$ is precisely $\Delta_j e$ in Eq. (22). Thus we have that

$$\frac{\partial \Delta_j e}{\partial b_j} = (I - \alpha W^{+j})^{-1} \left[\frac{1}{b_j} B \Delta_j b - \frac{\partial (I - \alpha W^{+j})}{\partial b_j} \Delta_j e \right]. \tag{23}$$

To assess the sign of such a derivative we compute $\Delta_j W e$. To do so note that

$$\Delta_j W = \begin{bmatrix} 0 & \frac{-b_2}{|G|(|G|-1)} & \cdots \frac{b_j}{|G|} \cdots & \frac{-b_k}{|G|(|G|-1)} & \cdots\cdots \\ \frac{-b_1}{|G|(|G|-1)} & 0 & \cdots \frac{b_j}{|G|} \cdots & \frac{-b_k}{|G|(|G|-1)} & \cdots\cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{b_1}{|G|} & \frac{b_2}{|G|} & \cdots 0 \cdots & \frac{b_k}{|G|} & \cdots\cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{-b_1}{|G|(|G|-1)} & \frac{-b_2}{|G|(|G|-1)} & \cdots 0 \cdots & \cdots & \cdots\cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{-b_1}{|G|(|G|-1)} & \frac{-b_2}{|G|(|G|-1)} & \cdots \frac{b_j}{|G|} \cdots & \cdots\cdots & 0 \end{bmatrix}.$$

Thus $[\Delta_j W e]' = \left[A_1, A_2 \ldots A_{|G|} \right]$ where for $i \neq j$, $A_i = -\frac{\sum_{m \neq i,j} b_m e_m}{|G|(|G|-1)}$ and $A_j = \frac{\sum_{m \neq j} b_m e_m}{|G|}$. Note then that $(\Delta_j e)_j > 0$, that is, individual j exerts positive effort in $G_1' \cup \{j\}$. Thus using Eq. (23) we conclude that

$$\frac{(\partial \Delta_j e)_i}{\partial b_j} > 0$$

for each $i \in G \cup \{j\}$, since $(I-\alpha W^{+j})^{-1}$ is a matrix of positive entries and $\left[\frac{1}{b_j} B\Delta_j b - \frac{\partial(I-\alpha W^{+j})}{\partial b_j}\Delta_j e\right]$ in Eq. (23) is a vector of positive entries, specifically, entry j is $(1+\beta)$ and each entry $i \neq j$ is $\alpha A_j/|G|$. ∎

Proof of Lemma 8. Consider the RD-partition described in Example 3 and let $b_i \geq \underline{b}(\alpha,\beta)$ for each $i \in \mathcal{I}_1$. For such a partition to be stable for each individual $j \in \mathcal{I}_2$ condition in (i) in Lemma 3 must hold. A sufficient condition for that to happen is that $e_{j,G_1} \geq 2\beta/\alpha$. We now consider the best reply of an individual j

$$e_{j,G_1} = b_j(1+\beta) + \alpha\left[\frac{\sum_{i\in\mathcal{I}_1} b_i e_{i,G_1'}}{|G_1|-1} + \frac{\sum_{i\neq j\in\mathcal{I}_2} b_i e_{i,G_1}}{|G_1|-1}\right]. \tag{24}$$

Since $b_i \geq \underline{b}(\alpha,\beta)$ for each $i \in \mathcal{I}_1$, each of these individuals exerts, at least, a level of effort β/α in G_1. Thus, a lower bound for Eq. (24) is

$$b_j(1+\beta) + \frac{\beta\sum_{i\in\mathcal{I}_1} b_i}{|G_1'|-1}.$$

Therefore a sufficient condition for (i) in Lemma 3 to hold for each $j \in \mathcal{I}_2$ is that

$$b_j(1+\beta) \geq \frac{\beta}{\alpha}\left[2 - \frac{\alpha\sum_{i\neq j\in\mathcal{I}_1} b_i}{|G_1|-1}\right].$$

If this expression holds for the least productive individual $k \in \mathcal{I}_2$, it holds for everyone in such a group.

∎

Proof of Lemma 9. The system of equations described in Eq. (15) has a unique solution if and only if $1 > \gamma_1(V)$ where $\gamma_1(V)$ is the largest eigenvalue of V. The proof is analogous to the one of Lemma 1 and hence omitted. It remains to show that for each pair $i, j \in G$ such that $b_i > b_j \Rightarrow e^E_{i,G} > e^E_{j,G}$. Let $\delta_1 = (|G|-1)^{-1}\sum_{k\neq i,j} b_k e^E_{k.G}$ and $\delta_2 = (|G|-1)^{-1}\sum_{k\neq i,j} e^E_{k.G}$ and thus

$$e^E_{j,G} = b_j(1+\alpha\delta_2) + \alpha\delta_1 + (|G|-1)^{-1}\alpha(b_j+b_i)e^E_{i,G},$$

and

$$e^E_{i,G} = b_i(1+\alpha\delta_2) + \alpha\delta_1 + (|G|-1)^{-1}\alpha(b_j+b_i)e^E_{j,G}.$$

We thus have that $[e^E_{i,G} - e^E_{j,G}][1+\alpha(|G|-1)^{-1}(b_i+b_j)] = (b_i-b_j)(1+\alpha\delta_2)$. As $b_j < b_i$, the right-hand side of this equality is positive, so is the left-hand side. That implies that $e^E_{i,G} > e^E_{j,G}$ and thus it directly holds that $b_i e^E_{i,G} > b_j e^E_{j,G}$. ∎

Proof of Proposition 3. Consider a group G and use the expression in Lemma 9 describing efficient efforts in matrix form. We have that

$$\frac{\partial e_G^E}{\partial b_i} = \frac{\partial (I-\alpha V)^{-1}}{\partial b_i} b + (I-\alpha V)^{-1} \frac{\partial b}{\partial b_i}. \tag{25}$$

In this expression, the second component is a vector of positive entries. Regarding the first component, we have that

$$\frac{\partial (I-\alpha V)^{-1}}{\partial b_i} = -(I-\alpha V)^{-1} \frac{\partial (I-\alpha V)}{\partial b_i} (I-\alpha V)^{-1}, \tag{26}$$

where for i and each $j \neq i$

$$-\left[\frac{\partial (I-\alpha V)}{\partial b_i}\right]_{ij} = -\left[\frac{\partial (I-\alpha V)}{\partial b_i}\right]_{ji} = \frac{\alpha}{|G|-1}, \tag{27}$$

and for $j,k \neq i$

$$\left[\frac{\partial (I-\alpha V)}{\partial b_i}\right]_{kj} = \left[\frac{\partial (I-\alpha V)}{\partial b_i}\right]_{jk} = 0. \tag{28}$$

Thus, $[\partial (I-\alpha V)^{-1}/\partial b_i]b$ in Eq. (25) results in a vector of positive entries. Therefore, $\partial e_G^E/\partial b_i$ in Eq. (25) has each entry positive. Using Eq. (25) it follows that

$$\frac{\partial}{\partial b_j}\left[\frac{\partial e_G^E}{\partial b_i}\right] = \frac{\partial}{\partial b_j}\left[\frac{\partial (I-\alpha V)^{-1}}{\partial b_i} b + (I-\alpha V)^{-1} \frac{\partial b}{\partial b_i}\right]. \tag{29}$$

Consider a group of cardinality higher than two and note that for each i, each entry in V includes the element b_j for each $j \neq i$, thus the second component in Eq. (29),

$$\frac{\partial}{\partial b_j}\left[(I-\alpha V)^{-1} \frac{\partial b}{\partial b_i}\right] = \frac{\partial (I-\alpha V)^{-1}}{\partial b_j} \frac{\partial b}{\partial b_i}, \tag{30}$$

is a vector of positive entries.[41] The first component in Eq. (29),

$$\frac{\partial}{\partial b_j}\left[\frac{\partial (I-\alpha V)^{-1}}{\partial b_i} b\right] = \frac{\partial^2 (I-\alpha V)^{-1}}{\partial b_i \partial b_j} b + \frac{\partial (I-\alpha V)^{-1}}{\partial b_i} \frac{\partial b}{\partial b_j},$$

is a vector of non-negative entries. Thus, for each pair of individuals $i,j \in G$ it follows that (i) $\partial e_G^E/\partial b_i > 0$ and (ii) $\partial/\partial b_j \left[\partial e_G^E/\partial b_i\right] > 0$.

We therefore conclude that (i) each individual's effort is increasing in others' productivity and (ii) the more productive each individual is, the more sensitive she is to an increase in

[41] The case of groups of cardinality two is analogous and the only difference is that the main diagonal of V^k is composed of zeros, for any value of k. Despite this fact, the above statement regarding the derivative in Eq. (30) follows.

others' productivity. By the proof of Proposition 2 aggregate utility of individuals within a group is essentially the sum of individuals' private product. Since private product is productivity times effort in an efficient partition non-singleton groups should be consecutive.

Finally, note that the sum of private products (divided by two) can be written as

$$2^{-1}\left[\sum_{i\in\mathcal{N}} b_i^2 + \alpha\sum_{G\in\mathcal{G}}\sum_{i\in G} b_i[b_i\overline{e}_{i,G}^E + A_{i,G}^E]\right].$$

Since for each individual i the efficient effort and others' average performance are increasing in productivities, a singleton group must consist of the least productive individual.

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